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Investigation of interstage buffering and its effects

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INVESTIGATION OF INTERSTAGE BUFFERING AND ITS EFFECTS

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ABSTRACT

While the process times of stages in a production line will vary from stage to stage about some distribution mean, delay time in the line can be minimized by decoupling the stages. The minimization in delay time, in conjunction with a minimum dollar investment can be accomplished through use of an optimum capacity in-process inventory buffer between production stages.

A simulation model was employed and through variation of line parameters the factors affecting buffer capacity were determined. Then, a relationship was developed for optimum buffer capacity versus output time per unit.

A functional relationship is presented for determining the optimum buffer capacity for a known output rate. The function is restricted to output rates of 24 units per hour or greater.

CHAPTER I

INTRODUCTION

It has long been recognized that the use of an in-process inventory between production stages of an assembly or manufacturing line is desirable, if not necessary. Methods for determining the optimal amount of in-process inventory required have not been stated without severe limitations being imposed. The factors making an in-process or buffer inventory of this nature necessary are varied. Some of these are: differences in length of process time between stages, machine down-time and an effect called the backlog syndrome (7). Any of these factors can cause delays in the processing times of units in a line which in turn results in idle machine and labor capacities. Since this idle capacity means a loss of time and money it's desirable to minimize the contributing factors.

One method of decreasing these delays is to include some type of buffering between each stage of production. The presence of this buffer will attempt to smooth out production and in turn reduce unit delays. This buffering action would, however, reduce only part of the delays mentioned. Specifically, the buffer should take care of differences in production times at various stages. It would be desirable to take care of machine down-times in the same manner but buffer capacities would be prohibitive due to holding costs.

The other delay mentioned, the backlog syndrone, is an effect due to a varying amount of work in front of the worker at a production stage. Gomersall (7) reports it to be a psychological factor due to the line personnel adjusting their rate of work, dependent upon the build-up or reduction of work to be completed. The effect is apparently due to the assumption on the operator's part that if there is no more work in front of his station he will remain idle until work is available. In other words, a worker adjusts his rate accordingly, performing the same amount of work over a large period and cut down on his idle time. In this case, the average output rate of the line remains constant, but the operator has less idle time. If a buffer stock is present in front of a machine, preceding production stage stoppage has a diminished effect on that machine. The operator will not have to sit idle the whole time the preceding stage is idle. In this manner his work speed will be more uniform.

Many attempts have been made to model a production line with interspersed buffer inventories but all have been of limited value. Some representative methods will be presented in Chapter II. These attempts have, in almost every case, been limited to a two-stage line with some particular distribution associated with the processing time, machine down-time, etc. Those models not limited to a two-stage line have utilized only the Poisson distribution which, in a practical case, does not describe conditions.

The prime reason for limited success in this area is that too many variables affect an analytic solution to the problem and some approach other than an analytical solution is indicated.

This paper will concern itself with the smoothing of production, taking into account only the variations in production times which, when provision for a buffer stock is allocated, may tend to reduce the effects of the backlog syndrome. Down-times of the stages will not be included in the investigation since these longer type delays would mask any effects due to the varying processing times.

The investigation procedure will be to determine effects of various line parameters on the production lines' output rate using normally distributed production times. Then, uniform and gamma distributions will be employed to demonstrate the effects of those production time distributions. Using the investigation results, a model will be developed which will be used for determining the optimal buffer capacity.

CHAPTER II

PREVIOUS ATTEMPTS IN THE AREA

There have been many attempts to solve the problem of whether two manufacturing stages should be separated by an inventory. Most of these methods have been made through the use of queuing theory and dynamic programming (10), (20). Using these approaches discussions have, in most cases, been limited to lines consisting of two stages because the analysis is dependent upon too large a number of variables to allow further analytic solution. The exception has been developed in general but a major problem exists in the development.

For the following discussion a production line will include continuous manufacturing and/or assembly operations. The line will consist of a series of stages which process material and pass it on to a succeeding stage for further processing. The line may also include storage units or bunkers.

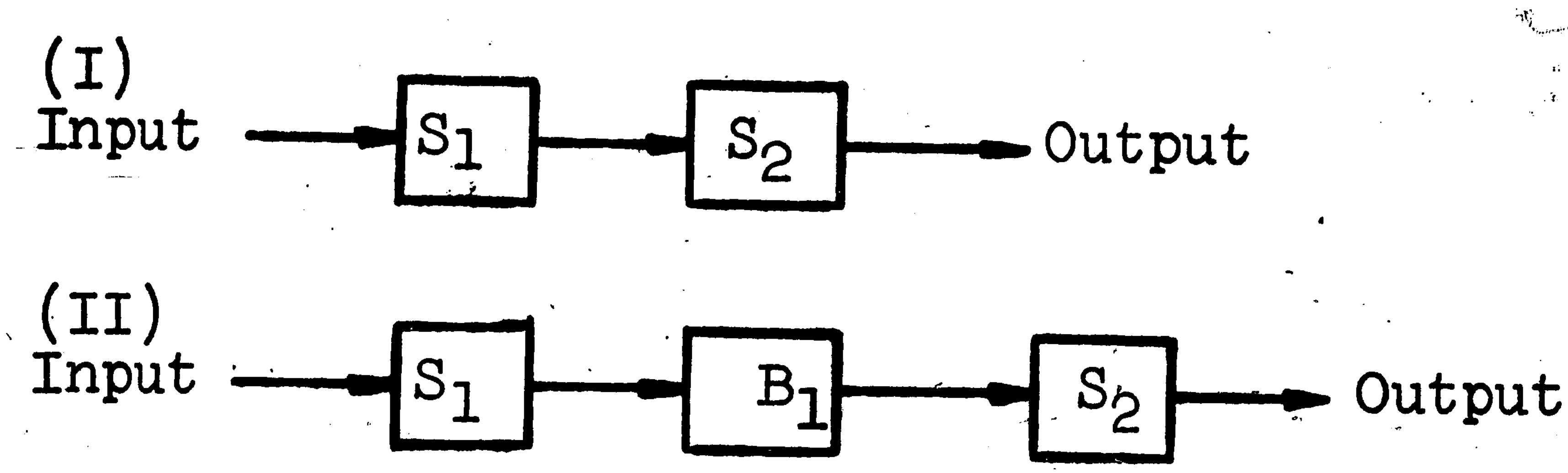

Fig. 2.1 Possible Line Configurations

In the preceding figure the line configurations consist of production stages either with or without a bunker between each production stage. Other configurations such as paralleling stages can be used but analysis would be similar to that for a single stage with a higher output rate.

In the following discussion several parameters which affect line production are used in the following context. Cycle time for a stage includes the time required to receive, process and move the product to the following bunker or production stage. The cycle time may be fixed or it may vary according to the unit being worked on, or the operation at that stage. Another parameter, called set-up time, can affect production efficiency and includes scheduled stoppages (preventative maintenance, replacing worn tools, replenishing raw materials, etc.) and unscheduled stoppages (down-time for repairs). The cycle time and set-up times have been found to be characterized, in general, by different distributions. The cycle time approximately follows a normal distribution (14) while the setting time can be approximated by an exponential distribution (14).

Straight-forward Approach

Referring to the simple line in Figure 2.1 (I), the following approach could be used. Without a bunker storage between manufacturing stages, the second stage must stop whenever the first machine is down, or if the first stage is still processing a unit after stage two has completed processing on its unit. Also, the first machine must stop if stage two is down or is still processing a unit after stage one has finished with its unit. Now, if p_1 and p_2 are the fractions of stop time on the first and second machines, respectively, the output (R) expressed as a fraction of the maximum possible output (both machines having the same mean processing time) is given by:

$$R = 1-P_1-P_2+P_1P_2$$

As an example: if $P_1=P_2=0.10$ then $R = 1-0.10-0.10+0.01=0.81$.

By extending this analysis it can be shown that output of a system is increased by providing bunker storages between each stage. Consider a two manufacturing stage system with an infinite capacity storage between stages. Also, assume an infinite capacity store in front of the line and at the output. Now, with these assumptions there will always be a supply of units going into each stage and also room for a unit after processing by a stage. Then, the output is limited by the worse stage (that with the greater down-time) and is given by:

$R=1-p$, where p is the fraction of down-time of the worse stage. In the previous example R would equal 0.90

so the efficiency of the line has increased with a gain in output of .09 or 11 per-cent of the previous output. This example does show an increase in efficiency using the bunker but only for an infinite size storage.

The work that has been accomplished in attempts by other authors can now be shown.

Loss Transfer Method

The first analysis of internal stores in a production line was presented by Vladziyevsky[1] and his method will be presented next.

The output interval (time between successive units of output) for the $i^{\underline{th}}$ stage, is given by: $(X_c)_i = \tau + t_i$ with the distribution function $f(X_c)_i$, where τ is the cycle time and t_i is the set-up time.

The mean output interval of the complete line must be greater than the maximum of the processing cycles of the stages, and $(X_c)_{mean} > (\tau_i)_{max}$ and the efficiency of the line is given by η, where

$$\eta = \frac{(\tau_i)_{max}}{(X_c)_{mean}}$$

Also, unit productivity of the line, Q, is given by:

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[1] Vladziyevsky's method is presented in papers by Koenigsberg (13). English translation not available.

$$Q = \frac{\phi}{(X_c)_{mean}} = \frac{\eta\phi}{(\tau_i)_{max}}$$

where ϕ is the time available for using the line.

The problem is: determine to what extent the productivity of a line depends on its separation into a larger or smaller number of successive sections connected to each other by bunkers capable of storing the in-process units or supplying input to the stage when there are no units being completed by the previous stage.

In this approach, only the feeding of the stage from a previous one is included. The model does not concern itself with storing of material from a stage if the succeeding stage is down for any reason.

In any line, there is a limit on the number of units to be allowed in a bunker storage. However, because the production times of each stage varies about some mean, the production for one stage will exceed that of the next stage at some point in time and at other times it will be less than that of the next stage. Here, a pulsating type of store is in use, because at times the stage will feed the store and then other times the stage will not but in each case the succeeding stage could still be drawing from that store. Therefore, the lower limit on the size of this store is zero and the upper limit is the allowable capacity, with the actual size being a variable.

The optimum capacity for buffers is given in two ways. If the criteria is to minimize cost the capacity would be:

$$n_{\theta} = \sqrt{\frac{B_o \cdot b(e-1)}{1+B_c(e-2)}}$$

$B_o = \sum_{i}^{n} B_i$ and B_i is loss of i^{th} stage due to set-ups.

$b = \frac{1-a^n}{1-a}$ where "a" is a loss transfer coefficient (portion of loss in the preceding stage which is transferred to succeeding stages).

B_c = set-up loss of a buffer.

$e = \frac{E_o}{E_c}$ where E_o is the cost per unit time of the line and E_c is cost per unit time of the buffer.

If the criteria were to maximize output rather than cost,

$$n_q = \sqrt{\frac{B_o}{B_c}(1+a)} \; .$$

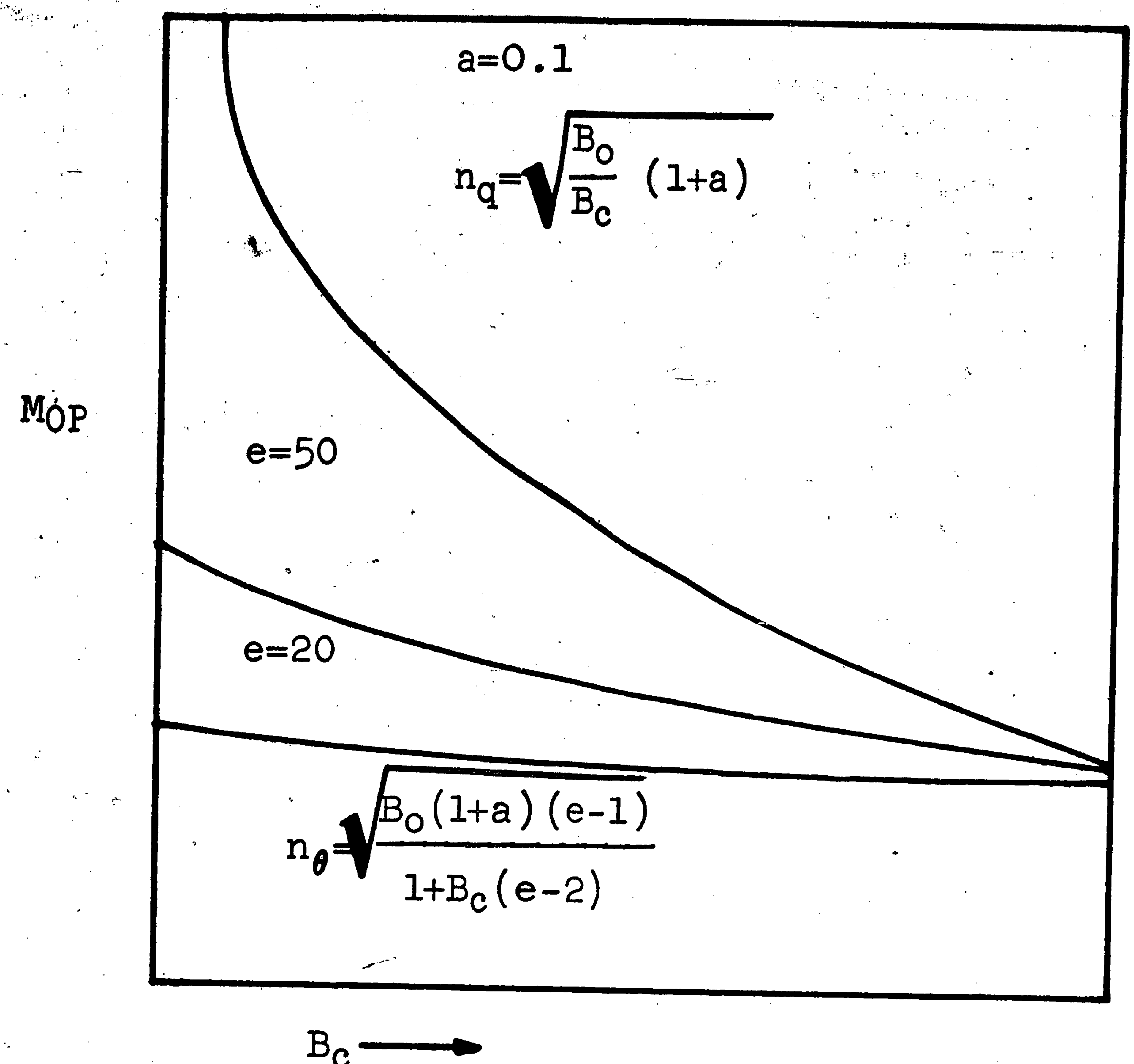


Figure 2.2

Relationships for optimum n are shown in the above figure. A comparison of the two general solutions gives

A summary of the above analysis is written below for the cost minimization criterion:

For $B_o= 0, B_c= 0$ ($\eta_o=1, \eta_c<1$) $\qquad n_\theta=0$

For $B_o=B_c=0$ ($\eta_o=\eta_c=1$) $\qquad n_\theta=0$

For $B_o\neq 0, B_c=0$ ($\eta_o=1, \eta_c=1$) $\qquad n_\theta=\dfrac{B_o b(e-1)}{1+B_c(e-2)}$

and $e = \frac{E_o}{E_c}$. The results for output maximization can be similarly expressed.

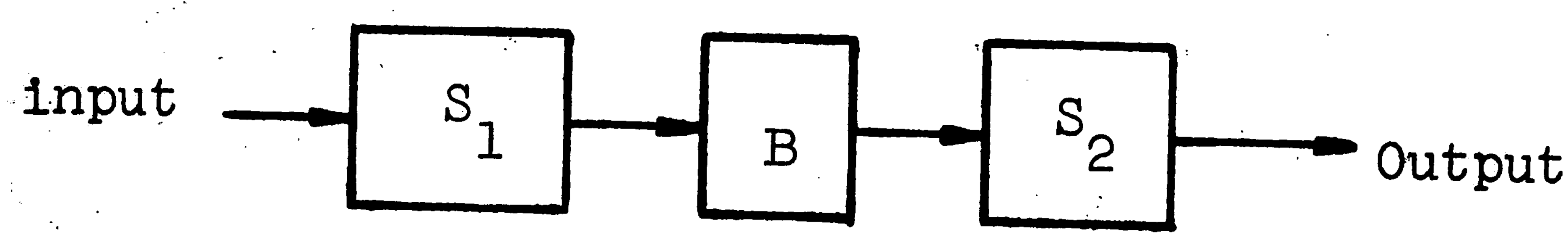
The second part of Vladziyevsky's solution is to determine the limiting size of the bunker stores. The value of the portion of loss in a stage which is passed to the next stage depends upon the mean level of the buffer and statistical parameters of the production stages in a line.

Now, while this method has been analytically extended to an n stage line it has a serious defect. The method can only take care of forward delays. These delays are caused by a succeeding stage being held up for lack of parts to work upon. Another delay (backward delay) must be considered in practical situations to take care of the case when a stage is stopped because a succeeding buffer is full.

Stochastic Method

While the previous method could be described as a probabilistic model considering forward transfer of losses, this next model used in studies by Finch (5) can be described as a stochastic model. This model includes the effects of stoppages due to breakdowns and set-ups, but is not restricted to the balanced line (all stages having the same process time and setting time distributions) assumption. No reference has been found in the literature which makes use of this model. The following discussion is limited to a two stage line connected by a bunker but could be extended to an n stage line with some difficulty. It should be noted that the number of state possibilities does increase rapidly.

Considering the simple line shown below, the buffer consisting of n units serves as both a supply source for the second stage, and a storage space for the output of the first stage.



$0 \leq \text{Capacity} \leq \infty$

Now, each stage must be in one of two states:

(a) working stage (denoted by "0")

(b) non-working state (denoted by "1").

The problem is then to determine the effect of buffer size on the output of the two stage system (assuming a steady input into stage 1 and infinite storage at the output of stage 2). At the same time, the optimum size of the buffer is desired.

The distributions of working and down-time will be approximated by exponential distributions, i.e., the probability that a working down state lasts at least t minutes is equal to ($e^{-\mu_i t}$) where $1/\lambda_i$ and $1/\mu_i$ are the mean working and down-time distributions respectively, for the i^{th} state.

Each stage of a line is characterized by the following parameters:

$l_i = 1/\lambda_i$ = mean working time duration

$m_i = 1/\mu_i$ = mean down-time duration

h_i = work rate

$1/h_i$ = work cycle time or process time

$u = \lambda/\mu$

The work cycle time is assumed constant.[1]

First, the probability that in a given state[2] the buffer contains n units when the system is in steady

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[1] In the case studied by Finch (automobile assembly line) this is a reasonable assumption.

[2] There are four possible states in a two stage system: 1(0,0) both stages working, 2.(1,0) first stage down, second stage working, 3.(0,1) first stage working, second stage down, 4.(1,1) both stages down.

state, and the probability distribution of the output during each of the states, must be calculated.

To calculate the gain of a line containing two stages separated by a bunker over a two stage line with no bunker, only the case of $h_1 \geq h_2$ need be considered when the first stage is down for set-up or repair. In the other cases, the output of the line will be the same for both systems. Let P_J be the output during the j^{th} (1,0) state (i.e., first stage down, second stage working) and g_k the output during the k^{th} (0,0) state; no line output when the second stage is down. During a time interval T the system will have undergone m-1 changes of state. It will have been in the state (0,0) m_1 times and in the state (1,0) m_2 times. The gain in output produced by a store of size n can be written as:

$$G(N) = u\left[1 - \frac{(1+u)\ (\mu_1+\mu_2)\ h}{h(1+u)\ (\mu_1+\mu_2)+N(\mu_1+\mu_2)\ (\mu_3+\mu_1)}\right]$$

when $\lambda_1/\mu_1 = \lambda_2/\mu_2$.

Queue Model

The production line discussions so far have been concerned with work cycle time (for a production state) which are fixed or have a limited variation. If the line is assumed to be a tandem servicing station with each section providing service at a mean rate μ with exponential service time, then we have a system which can be described in a mathematical sense as a queueing system.

Consider a production line of the form in Figure 2.1 (II). The cycle time of each stage (cycle time includes production and set-up) is assumed exponentially distributed. Also, it's assumed that material enters the line at a mean rate λ which has a Poissom distribution, i.e., the probability that n units enter the system in time interval T is:

$P_n = \frac{\lambda^n}{n!} e^{-\lambda}$ where λ is the mean number that enters in the time interval. This type of line has been treated by Jackson (12) and Hunt (11). Also, Burke (2) has shown that if the input to the first stage is Poisson, then the steady state distribution of the number of service completions in an arbitrary time interval is the same as the input distribution. If the buffer between stages is infinite, the input to the second stage is also Poisson. In this case the relationships already developed for simple queue systems can be used and the results extended to tandem systems. From these results, the required in-process storage can be evaluated to produce a given output for the system.

If $p_i = \lambda/\mu_i$ where p_i is the service factor or utilization factor of the i^{th} stage, the following results are obtained:

$p_o = 1 - p_i$ = probability that no units are in the stage

$\bar{w}_i = \dfrac{p_i^2}{1-p_i}$ = average number of parts waiting in front of the stage.

${}^{2}_{w} = \overline{w_i^2} - \bar{w}_i^2 = \dfrac{p_i^3}{1-p_i}$ = variance of number of units waiting in front of the stage.

It's assumed that units are accepted by a stage one at a time, in the order of arrival and that μ_1 is the acceptance rate.

The probability that the queue will be of length n is

$$P_n = p^n(1-p)$$

and the desired confidence interval can be obtained by summing the probabilities in the tail of the distribution and finding n such that

$$\sum_{n}^{\infty} P_n \leq .05 \text{ or } .01.$$

The above equations are first approximations to the correct values for the distributions and have used the results which queueing theory gives when an infinite store is available. In a practical situation the buffer would be some finite capacity.

Hunt (11) has discussed several examples of sequential arrays of waiting lines which can be considered as production lines. His four cases are broken down as follows:

Case 1: Infinite queues in front of each stage (infinite bunkers).

Case 2: No queues allowed except in front of the first stage where an infinite store is allowed.

Case 3: Finite queues in front of each stage except in front of the first stage where an infinite store is allowed.

Case 4: No queues and no vacant facilities are allowed except that the first stage may have an infinite queue (conveyor line with no bunkers).

Hunt's study was concerned primarily with evaluation of the mean number of units in the system, $\bar{n}$, and maximum possible utilization, p max.

Case 1 is the standard for comparison and represents the lowest possible value of $\bar{n}$ for a given utilization. Case 2 differs very little from Case 1 for mean number of units in the system but is markedly different if the maximum possible utilization is concerned. Since, in Case 4, no empty stages are allowed, the mean number of units in the system approaches Case 2 as the utilization decreases. The mean number of units in the system for Case 3 should lie between those for Cases 1 and 2.

Case 3, the more interesting from a practical sized bunker storage viewpoint, increases the maximum possible utilization with increasing bunker capacity and approaches unity. As the number of stages increases more storage is required between stages to obtain an equal utilization for that of a two stage system.

The data supplied by Hunt is shown in the following figure with only the number of units in the queue being of interest.

In a practical case, the assumption of a Poisson input may not be a good one since in many cases the input is fixed or subject to a smaller variation than that characterized by the Poisson distribution. Nonetheless, it can serve to provide reasonable quantitative data.

Case 3 Only Length of Queue	Number of Stages 1	2	3
2	1.0	0.75	0.6705
3	1.0	0.80	
4	1.0	0.833	
8	1.0	0.90	
18	1.0	0.95	

Figure 2.3 Calculated Values of p max, the Maximum Possible Utilization.

Comparison of Results

Because of different assumptions made in the models discussed any direct comparison is difficult. The queueing model assumes a random input arrival to the line, with a random exponential service, while the others assume a constant arrival rate and service times which are a constant plus some exponentially distributed variation about that constant. The constant arrival rate to the line assures that there will always be a unit available to the first stage while with the random arrival this is not assumed.

Also, comparison made must only be for a two stage system since the stochastic and queueing models get much too complicated for longer lines. These methods were presented to give some idea of work which has been done. As can be seen, even for the investigation of two stage systems previous work has been very limited with very restrictive assumptions being made.

The loss transfer method assumes that the loss of a line is the sum of the losses of the individual stages. However, when several stages are down simultaneously there is only one loss. The loss transfer model further assumes that the distribution of the number of units in the bunker is symmetrical. In the queueing model with exponential service or the stochastic model this is not an assumption.

The queueing model doesn't take down-time into account as presented, but it could be added by considering the service time as an overall output interval.

One factor that makes the queueing model an interesting one is the fact that it's the only model of those presented that includes not only delays from bunkers being empty, but also from bunkers being full. Since both factors are important in the practical case, a model should include both types of delays.

The only model of those presented which could be easily extended to an n stage production line, is the queueing model. However, the assumption of exponential arrivals and service times is too restrictive to be of practical value. The other two models, as presented cannot be extended easily or accurately because of the numerous existing solution possibilities. In the stochastic model the state possibilities increase by the function 3^n where n is the number of stages in the line. For a 3 stage line there are 27 state possibilities to examine.

The simulation model to be used in a later chapter is an extension of Vladziyėvsky's Loss Transfer Method, taking into account the forward and back delays. A completely analytical solution is not attempted and the approach then allows a generalization to an n-stage production line.

CHAPTER III

HYPOTHESIS AND ASSUMPTIONS

Hypothesis

It has been proven that an infinite capacity buffer storage between production stages does decrease delay time in a line. It remains to be shown that a smaller capacity buffer will also decrease delay time. The hypothesis of this paper is that the presence of a finite buffer capacity between stages will decrease delay time, thereby increasing unit output per unit time. Also, general rules for determining a minimum buffer capacity will be investigated.

To accomplish the above purpose a simulation program (to be described in Chapter IV) was written for the simulation of actual production line operation. Then, by varying parameters of the line various effects can be demonstrated. The purpose of this investigation is two-fold. First, effects of varying production time means and variances will be investigated. Then, production time distributions other than the normal will be used to determine whether any of the effects can be stated in general.

The criteria for determining an optimum capacity buffer will be unit delay time in the line. The minimum capacity required to realize the unit delay time will be assumed the optimum capacity. After this value is determined, further increase in capacity should not significantly decrease unit delay time but a decrease in capacity should result in a significant delay time increase. A one-tailed, t-test will be used to evaluate the resulting delay times.

Assumptions

The following assumptions and initial conditions were made to allow easier formulation of the model, but should be representative of conditions found for the majority of production lines.

A. Simplifying assumptions:

1. All bunkers are started empty in the simulation program.
2. An infinite size storage is present in front of stage one for supplying units to the line.
3. An infinite size storage is present at the output of the line to supply storage for units finished by the line.

B. Assumptions made in testing of parameters:

1. Balanced line--Each stage of the line has the same mean production time.
2. All buffers will be of equal capacity.

3. Buffers will be present between each stage of production.

As mentioned above, these assumptions are not necessary for operation or for use of the model. As will be seen in the description in later chapters, these factors could be changed.

The normal distribution was selected for representation of production times in the initial simulations. As shown by Lind (14) this distribution is a good approximation for the production times. The actual distributions can vary from approximately a normal distribution to a Pearson Type II or Type III. Therefore, after initial runs have been made to investigate parameter effects, the gamma distribution will be used to give a skewed representation of times. In the production process there is usually a lower limit on the actual time but the upper limit is not equally displaced from the mean. The upper limit is actually displaced further from the mean and a skewed distribution is then applicable.

One other distribution will be used to demonstrate some properties in general. The uniform distribution, while not a good representation of production times, should suffice to prove, with limits on the range of the distribution, certain parameter properties of the simulated line.

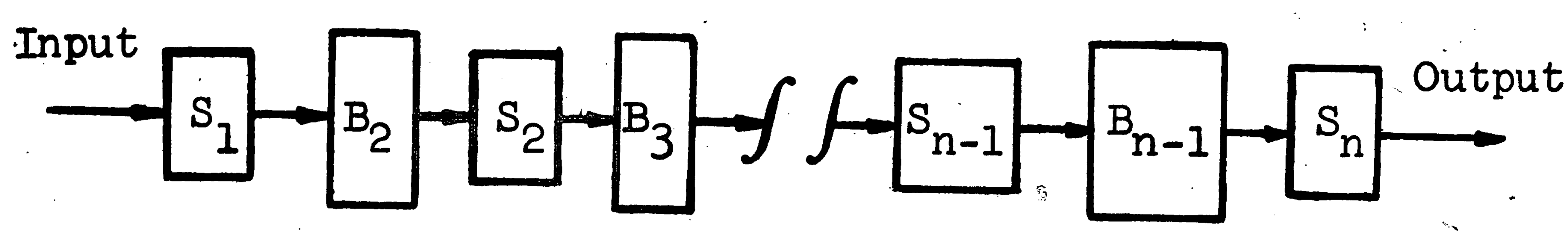
CHAPTER IV

EXPERIMENTAL PROCEDURES

To test the hypothesis presented in the previous chapter, a simulation program was developed to describe the production line with buffers between each production stage. Various parameters were then varied and the results are shown in Appendix D.

Next, different distributions were used in the simulation program to determine any general effects.

The line to be simulated is described in the following figure.



S_i - Production stage

B_i - Bunker storage

Figure 4.1 Line to be simulated

The parameter causing delays in the model is the difference in production rates between stages. Two types of delays are involved: a forward delay and a back delay. Both are due to the variation of the production times about some mean.

Simulation Program

To simulate the operation of a production line it is necessary for a simulator to include certain properties of that line. For the simulation of a line with bunker stores, the following functions must be performed:

1. Keep a sum of time used to produce present number of units through the line.
2. Record the current status of all bunker storages.
3. Determine the first non-empty bunker.
4. Update the bunker storages as a production stage finishes processing a unit.
5. Compute a delay time when a production stage is delayed due to a bunker being full or empty and the next stage not being able to pass on a completed unit.
6. If the preceding bunker is not empty, decrease that bunker by one unit and note that the stage is busy processing. If the bunker is empty, the stage is noted as not processing. Then when the previous stage finishes a unit this stage can receive that unit immediately. Until then, however, the stage has been delayed.
7. If stages are stopped due to either full or empty bunkers, these stages will begin operation again only when either a unit becomes available

or a buffer is no longer full.

8. As a bunker completes a unit and accepts the next unit a new process time is selected from the distribution and added to the present total time.

9. The next available production stage is determined from a timetable look-up.

10. As the nth stage completes a unit, add one to finished units.

11. Repeat the previous steps until the finished units equal the desired number to be processed.

The following definitions of terms are used in the analysis:

(1) $T_{n,i} = P_{n,i}+I_{n,i}+D_{n,i}$

where:

$P_{n,i}$ = Production or service time for ith unit at nth stage.

$I_{n,i}$ = Idle time of nth stage waiting for ith unit to arrive at nth stage.

$D_{n,i}$ = Delay at nth stage experienced by ith unit waiting to move to nth bunker stage.

$T_{n,i}$ = Total time required for stage n to process the ith item.

$P(E_n^i)$ = Probability that the nth bunker stage is empty at the time unit i moves out of the nth production stage.

$P(F_{n+1}^i)$ = Probability that the n+1 bunker stage is full at the time production stage n has finished processing item i.

N = Total number of production stages in the line.

Now,

(2) $I_{n,i} = [T_{n-1,i} - P_{n,i-1}] \cdot P(E_n^i)$

Equation (2) represents a forward delay and results from the process time of the n-1 stage for the ith item, which is a longer period of time than the production time of the nth stage for the i-1 unit,[1] and the probability that the nth bunker will be empty at the time the i-1 unit moves into the nth bunker. The stage is idle until the n-1 stage has finished processing the ith unit.

Also, (3) $D_{n,i} = [T_{n+1,i-1} - P_{n,i}] \cdot P(F_{n+1}^i)$.

Equation (3) represents a back delay and results from the process time of the n+1 stage for the i-1 unit being greater than the production time of the nth stage for the ith unit, and the probability that the n+1 bunker will be full at the time the ith unit tries to move into that bunker. The nth stage must then sit idle, holding the ith unit until stage n+1 finishes the processing of unit i-1.

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[1]The n-1 stage is processing the ith unit and the nth stage is processing the i-1 unit.

Now, letting $R_{n,i} = P_{n,i}+D_{n,i}$, we have from (1) and and (2):

$$(4) \quad I_{n,i} = [R_{n-1,i}+I_{n-1,i}-P_{n,i-1}] \cdot P(E_n^i)$$

but we also know that

$$(5) \quad I_{n-1,i} = [T_{n-2,i}-P_{n-1,i-1}] \cdot P(E_{n-1}^i)$$

since the same conditions exist for the n-1st stage of production as in the nth stage.

Equation (6) shows the relation of the variables involved with $T_{n,i}$ being the total time for stage n to process item i. To find the total delay time at that stage, it is necessary to subtract from this total time, the actual production time $P_{n,i}$. Appendix B shows the full development of the equation in closed form.

$$(6) \quad T_{n,i} = R_{n,i}+I_{n,i}$$

$$\text{where} \quad I_{n,i} = \left[\sum_{J=1}^{n-1} (P_{n-J,i}-P_{n-J+1,i-1}) \cdot \prod_{L=1}^{J} P(E_{n-L+1}^{i})\right], \text{ and}$$

$$R_{n,i} = P_{n,i}+\left[\sum_{J=n}^{N-1} (P_{J+1,i-J-1+n}-P_{J,i-J+n}) \cdot \prod_{L=n}^{J} P(F_{L+1}^{i-L+n})\right]$$

$$+\left[\sum_{J=n}^{N} I_{J+1,i-J+n-1} \cdot \prod_{L=n}^{J} P(F_{L+1}^{i-L+n})\right].$$

The two parameters, probability of the nth bunker being empty at time i, $(P(E_n^i))$, and probability of the nth bunker being full at time i, $(P(F_n^i))$, are functions, not only of the previous stage or following stage, but also of each preceding stage or following one. The probabilities depend upon the length of time to process each unit before processing the ith unit since the times vary about the same mean time. Because of this fact a direct analytical solution is not practical. The probability terms, even if a mathematical representation were possible, would involve too many variables to be of practical value. Therefore, based on the previous analysis a simulation program was written which will perform the above-described functions.

The simulation program written for this experiment is described in the flowchart (Appendix A). As can be seen, the limit on size of bunkers must be fixed for a specific simulation. It is not necessary that all bunker storages be the same capacity.

Next, a production time distribution must be assumed, although the simulator is not limited to any particular distribution. Manufacturing processes, whether machine or operator controlled, show some inherent variation about their mean production rates varying from approximately normal (14) to positively skewed distributions similar to the Pearson Type III curve. Therefore, the normal was used and if down-times had been included they would be described by an exponential distribution.

To simulate the operation of a production line, the program operates in the following manner:

1. Process times for each stage in the line are selected from a normal distribution generator.
2. Add those times to the sum of all previous operation times.
3. Determine which stage finishes processing a unit first (a variable clock is used for timing of the line operations).
4. Current status of preceding and succeeding bunker storages are checked to determine whether or not this unit can be moved and if another unit can be brought into this stage for processing.
5. If the succeeding bunker is not full, add a unit to that bunker. If full, hold unit at stage.

To show the operation of the simulation program in processing and storing units, the following discussion will perform the operations manually for a two stage line separated by one bunker with a maximum size of one unit.

Initially the bunker is empty and both production stages have units to process. Figure 4.2 is a diagram of the following process.

1. Select processing times from a distribution:

(a) First stage - 1.2 minutes

(b) Second stage - 1.4 minutes

2. Determine stage finishing first (stage 1).

3. The storage is empty, so the unit from the first is added to the bunker.

4. Select a new processing time for production stage one - 1.3 minutes.

5. Determine next stage to complete a unit (stage 2).

6. Add one to finished parts counter.

7. The bunker has a unit, so decrease bunker by one unit and select process time for stage 2 - 0.8 minutes.

8. Determine next stage to complete processing - stage 2.

9. Bunker is empty so stage 2 is delayed until stage 1 completes processing 0.5 minutes later.

10. At 2.5 minutes, stage 1 will have completed processing its unit and because the bunker is empty with stage 2 not busy, the unit will be passed directly to stage 2, through the bunker and new process times selected for each stage.

The above process is completed with delays required if units aren't available in bunkers or previous stages.

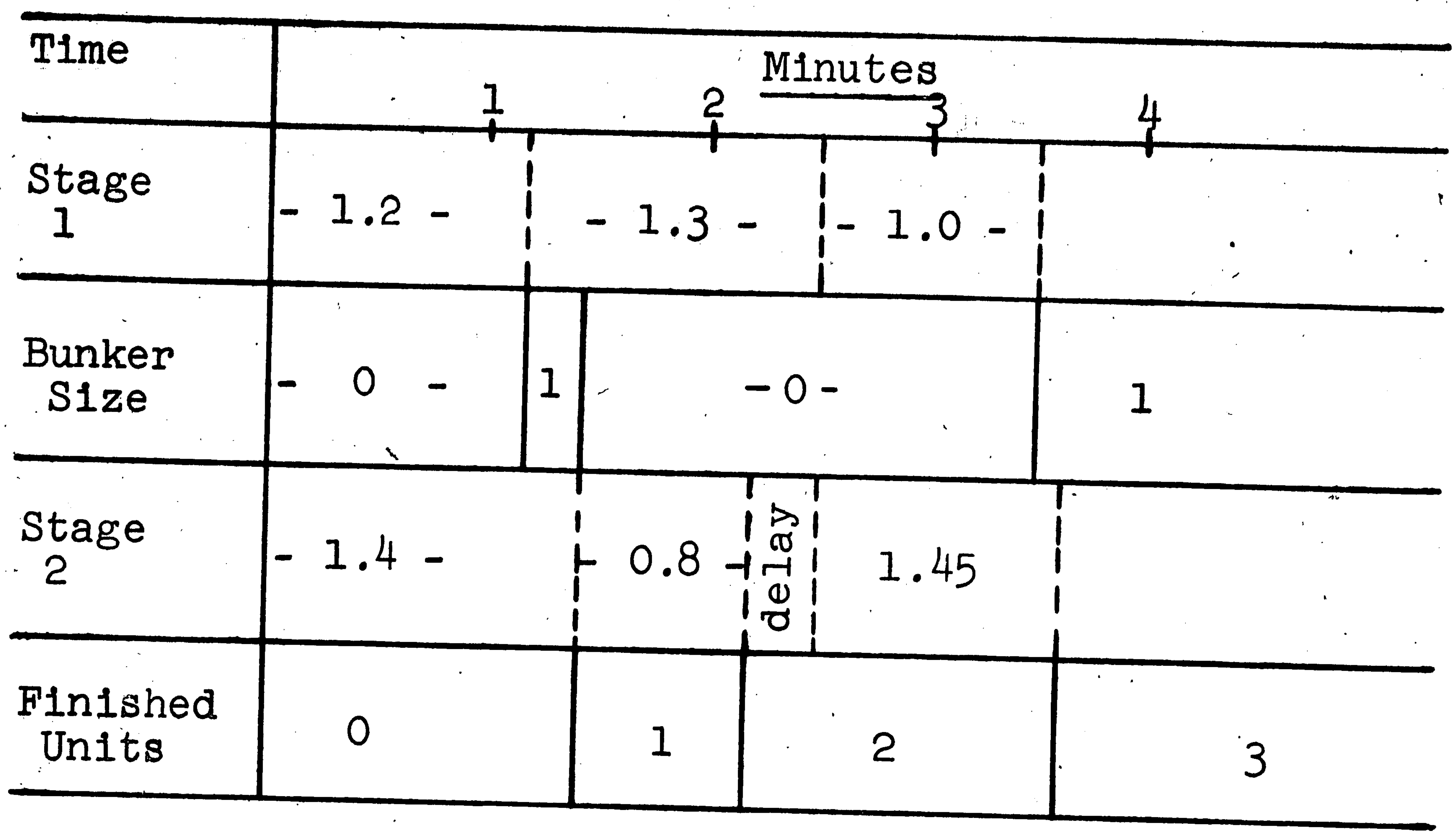

Figure 4.2 Sequential cycles to show operation of a line with Bunker Storage

The simulator was run for lines with 5, 6 and 10 stages. For each line length, the mean of the normal distribution was varied from 2.5 minutes to 10 minutes with standard deviations of 0.5, 1.0, 1.5, 2.0, and 3.0 for each mean value.

After these initial runs using the normal distribution, the uniform distribution was used with the same means and the ranges equaling up to plus and minus half the mean. The results obtained will be discussed in Chapter V.

CHAPTER V

DISCUSSION OF RESULTS

Determination of the Steady State Condition

Before any simulation runs were made it was necessary to determine that the production line had arrived at a steady state operating condition. This is a necessary requirement if a production line's true operating conditions are to be simulated. Until the line reaches steady state the true effects of buffer stages would not be realized. The output rate of the line will start at some value and then as the smoothing due to buffering becomes effective the output rate increases. The problem is then to determine how many units must be processed by the line before steady state is reached.

To detect arrival at the steady state condition for the production line, control charts were used. While the most common use of control charts is to monitor a process which is already in control they can be applied to other situations. In the application to be described it is necessary to determine when the output rate is "under control" or has reached the steady state output rate.

The output time per unit (reciprocal of output rate) of the production line was used as input to an $\overline{X}$ and R chart program. To decrease the number of input data points to the $\overline{X}$ and R program the output time was sampled after every 10 units of finished output. Sample groups of 3 data points were used to give an indication of fluctuation between successive samples of output rate. The $\bar{X}$ chart program output gives a plot of the output rate and shows the output time per unit converging to some steady state value. The range (R) portion of the program gives a plot of the difference between the highest and lowest value within the subgroups of 3 data points. By using this output in conjunction with the $\bar{X}$ plot the number of units to reach steady state can be determined. As a further determination of whether the process has reached steady state the control chart program was used with input data starting at that point where steady state was indicated from the initial runs. From this point an additional sample of 5000 units was finished by the simulated line, with periodic sampling of the output as before to make certain the process remains under control (steady state). It was noted that once the steady state output had been reached, fluctuation of output time data was within control limits.

Using the above-described procedure, the number of units to reach steady state was determined for various combinations of means and standard deviations for the normal distribution of the production times. The primary contribution to differing lengths of time to reach steady state came from the standard deviation. Appendix C contains a plot of units required to reach steady state, versus the buffer storage capacity. The plots are made for varying standard deviation. From these graphs it will be noted that the maximum number of finished units for reaching the steady state was 720 units. For ease of programming and to further insure a steady state condition all future runs were made with output sampling started at 1000 units and running for 3000 more units. Using a normal distribution to simulate production times a good estimate of output time per finished unit is then obtained.

General Effects of Parameter Variation

The simulation program was run with variation of 4 parameters as shown below:

	PROCESS TIME			
	MEAN (Minutes)	Standard Deviation	Number of Stages in The Line	Capacity of Buffer Storage
1.	2.5	0.5	5	0
2.	5.0	1.0	6	1
3.	7.5	1.5	10	2
4.	10.0	2.0		3
5.		3.0		4
6.				5
7.				10

Figure 5.1 Parameter Values

Each value of a line parameter was run with every value of the other parameters. In all, 420 runs were made. This number of runs represents 30 hours of IBM 360/50 computer time or over 4 minutes per run. Appendix D contains 4 graphs showing the effects on output time in excess of the mean time per unit for values of the various parameters. This time in excess of the mean can be directly attributed to a delay time per unit as described in Chapter IV.[1] As these graphs show, the curves are almost parallel and differ only in relative level as a function of the standard deviation and number of stages in a line. The most noticeable difference between curves is directly attributable to the standard deviation, and for a constant standard deviation, varying the other parameters produces a family of curves with little variation within the family. However, the standard deviation changes the level of the curves drastically. As can be seen on graph 1, with a mean of 2.5 minutes, when the standard deviation became a large percentage of the mean the curves are raised to a high level for small buffer capacities.

It will also be noted that for all combinations of parameters the output time in excess of the mean appears

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[1] Chapter IV(equation 2) forward delay:

$$I_{n,i} = [T_{n-1,i} - P_{n,i-1}] \cdot P(E_n^i).$$

(equation 3) back delay:

$$D_{n,i} = [T_{n+1,i-1} - P_{n,i}] \cdot P(F_{n+1}^i).$$

to approach some equilibrium condition, differing as a function of the standard deviation. The variation due to the number of stages has all but disappeared except when the standard deviation is of a magnitude equal to the mean. Even for these large variations about the mean it can be seen that the curve converges to some value with a bunker of 10 unit capacity.

Effects of Production Line Length on Delay Time

When a comparison of curves for the various means is made it becomes apparent that delay time increases as a function of line length. This result is to be expected since the number of times the last stage in the line is delayed is a function of the condition of the preceding buffer storage. This buffer will be empty, causing delays, whenever the next to last stage falls behind the last stage by enough time to deplete the storage between the stages. Since the output rate of a stage is dependent upon the condition of the storage preceding it, with more stages in a production line there are more buffer storages and production stages to affect the last stage. Therefore, the more delays in a line the lower output and efficiency of the line. Efficiency here is defined as the ratio of mean distribution time to actual output mean time. As the storage capacity is increased to 2-3 or more units the above-described situation has changed. Now the length of the line has small effect on delay time in the line and the delay times become approximately equal.

While some of the effects described above were suspected beforehand and logical explanations found, the extremely small number of units needed to provide a buffering action was not expected. This capacity of a buffer does not imply a constant level of units in a buffer since each stage has a distribution for the production times. However, having a capacity greater than zero does allow for variations between stages. Then unless standard deviations are very large only small buffers are necessary to smooth the effects due to varying production times.

Effects of Varying the Standard Deviation

As the standard deviation is increased for the various simulation runs it becomes apparent that the standard deviation has a great effect on the output units/time. In fact, as the curves in each of the 4 graphs are examined it will be noted that some general results can be stated.

When the ratio of standard deviation to the mean (σ/μ) is less than 0.5 a buffer capacity of three units between each production stage decreases delay time per unit to a minimum. This fact describes all values of the mean used up to this point. Any further increase in buffer capacity will not decrease delay time in the line.

When the ratio of standard deviation to the mean exceeds 0.5 the previously discussed rule doesn't apply. For these cases, as seen in the graphs (Appendix D) the curves do not level out with a buffer capacity of three units. In this case a capacity of five or more units is required. In the practical case, however, a ratio of more than 0.5 would not be realized since the 2σ limits would produce negative times at the lower end of this distribution. A ratio of 0.25 or less is probably the more realistic value.

Extension of Previous Findings

From the previously described analysis it can be seen that the length of a line, while having some effect on the minimum value of delay time in a production line, has very little effect on the required buffer capacity. Since the next step in the investigation is to use production times of lower values (higher output rate) it is desirable to decrease the number of runs required. For some initial runs, line lengths of 5 and 10 stages were used but as seen with lower output rates, only the minimum delay time is affected. Since this investigation is concerned primarily with buffer capacity a decision was made to carry out further simulations using only a line length of 5 stages. When distributions of production times other than the normal were used the number of production stages were again varied but there was no increase in required buffer capacity.

The area of investigation was extended to an output time per unit of 0.05 minutes (600 units per hour). The following parameter values were used.

	PROCESS TIME	
	MEAN (MINUTES)	STANDARD DEVIATION
1.	0.02	0.01
2.	0.20	0.10
3.	0.60	0.30
4.	1.00	0.50

The four parameter values for the production times were used with each buffer capacity.

BUFFER CAPACITY (UNITS)
5
10
20
30
40
50
60

Figure 5.2 Normal distribution parameter values.

A graph extending the area of coverage to the previously indicated values of the mean are shown in Appendix E. From these curves it can be seen that, depending upon the value of output time per unit, the buffer capacity needed to reduce delay time will vary. As the output time per unit decreases (output rate increases) the buffer capacity necessary for smoothing production will increase. This is a reasonable relationship since, logically it should take more units of buffering for an output rate of 600 units per hour than for 6 units per hour.

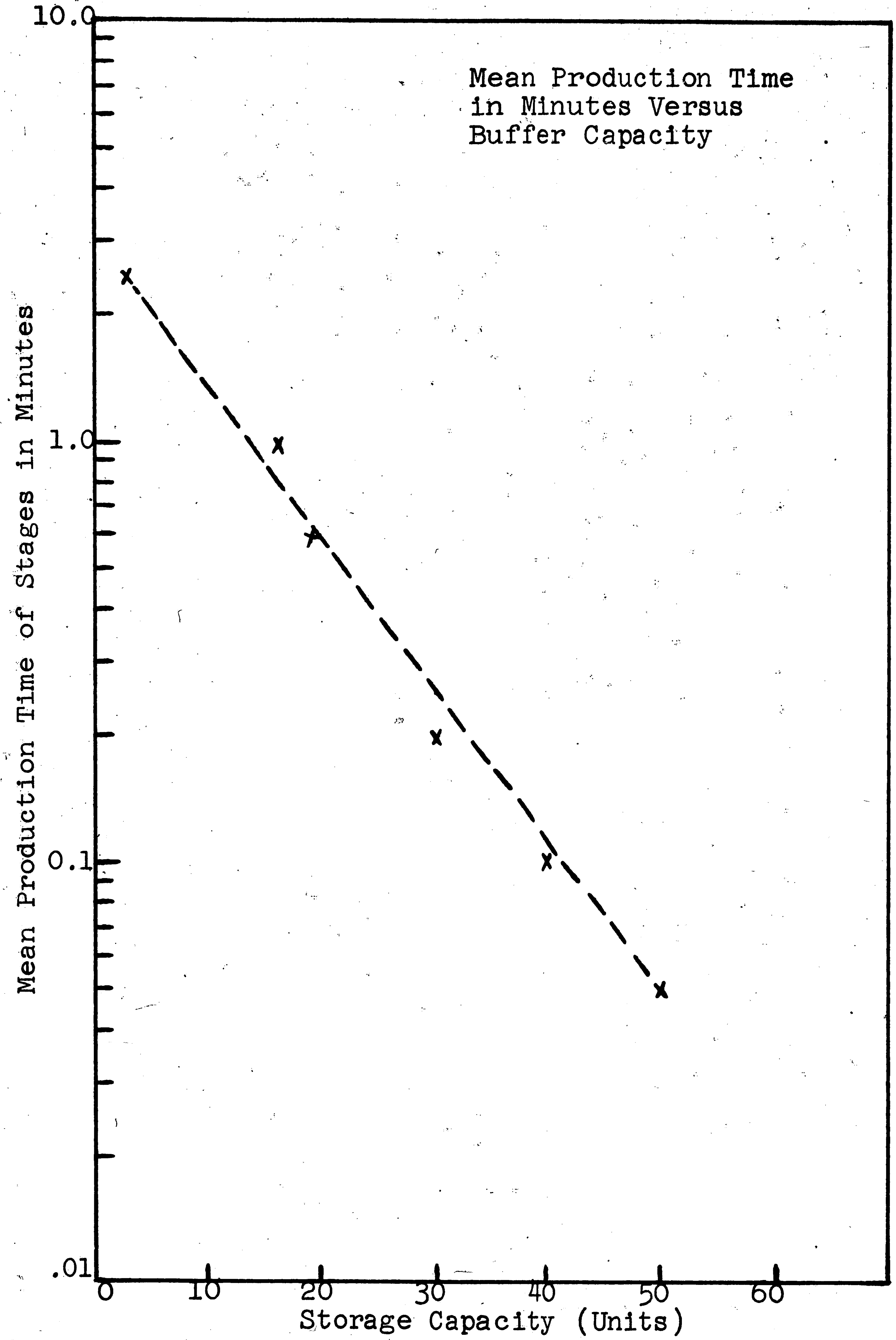


Figure 5.3 Required Buffer Capacity

To make further statements about buffer capacities required to reach the minimum delay time it is necessary to define the point on the curve which is considered the minimum value. For my work it was defined as the point at which the delay time equals 1.5 times the minimum value found on the curve. Since the curve in all cases converges to some value this seems to be a reasonable assumption. If a higher value of delay time is considered reasonable, the results will still be applicable. However, the value of buffer capacity would be lower in all cases.

After it was concluded that buffer capacity does increase with a decrease of output time per unit, the graph in Figure 5.3 was made. This graph shows the output time per unit versus buffer capacity required to reach the minimum delay time as described above. With this curve plotted on semi-log paper the data points follow an almost straight line. A linear regression program was used to determine the best fit for the data. The following figure shows the linear regression results with the equation of the line describing the data. The input to the linear regression program was ln y rather than y to give the relationship $y = Ae^{-Bx}$. Also, ln A was calculated by the program and necessary conversion was made to give the value of A.

Linear Regression Analysis

$\ln y = \ln A - Bx$, $y = Ae^{-Bx}$

B=-0.086
A= 3.251
R=-0.9953 (COEFFICIENT OF CORRELATION)
STD lnA=0.128 (STANDARD DEVIATION OF A)
STD B=0.0041 (STANDARD DEVIATION OF B)

ANOVA

SOURCE OF VARIATION	SUM OF SQUARES	DOF	MEAN SQUARE
DUE TO REGRESSION	10.8177	1	10.8177
ABOUT THE REGRESSION	0.1028	4	0.0257
TOTAL	10.9205	5	

Figure 5.4 Linear Regression Results

Extension to Other Distributions of Production Times

Since the correlation between buffer capacity and output time per unit is high it becomes necessary to know how the production time distribution effects the line. If changing the distribution of times will produce the same results as noted for the normal distribution general observations can then be made about buffer capacities. The gamma and uniform distributions were used in further simulations. With the samples used a range of density functions has been covered, and general statements can be made.

Results of using the various distributions are shown on graphs in Appendix E. As will be noted the same type of curves have resulted as those shown for the normal distribution.

With the above curves demonstrating the same effect as those discussed earlier it was determined, through use of an analysis of variance, that the distribution of production times does not have a significant effect on buffer capacity required. The ANOVA table is shown in the following figure along with the data used for the analysis.

	DISTRIBUTIONS		
Mean	Normal	Uniform	Gamma
1. 0.1	40	41	39
2. 0.3	27	26	29
3. 0.4	23	21	23
4. 0.6	19	18	18

ANOVA TABLE

Source of Variation	Sum of Squares	Degrees of Freedom	Mean Squares	F Ratio
Mean (A)	798.000	3	266.000	187.8
Distribution (B)	1.500	2	0.750	0.529*
Interaction term (AB)	8.500	6	1.417	
TOTAL	808.000	11		

*Not significant

Figure 5.5 ANOVA Table to show effect of production time distributions

The F ratio for the 99% confidence interval is $F_{2,6} = 10.92$ and for 95% confidence is 5.14. The observed ratio is 0.529 and is not significant. It can then be concluded that the distribution of production times does not have a significant effect on the required buffer capacity. It then follows that the equation:

$$y = 3.251\ e^{-0.086x}$$

y = Output time per unit of production

x = Required buffer capacity

will, in general, provide the required buffer capacity for any given output rate which equals the reciprocal of y (output time).

Effect of Large Output Time Per Unit

While the buffer capacity is inversely proportional to the output time per unit as shown on the graph (Figure 5.3), there is a lower limit for buffer capacities. With an output time per unit greater than 2.5 minutes the optimum capacity approaches unity. This fact restricts use of the model to line output rates of over 24 units per hour and its usefulness will primarily be in machine processes. It should be noted that for any output rate, some buffering will reduce unit delay time and decouple the production stages. To determine when a buffer should be provided for decoupling production stages, an economic evaluation must be made.

Testing of Formulation

To determine whether the function $y = Ae^{-Bx}$, with proper constants, does give the optimum value for buffer size a test was made. For this test 10 mean values were picked at random from a uniform distribution (2.5 minutes to 0.05 minutes). Using the above function a buffer capacity was determined. These values were then used in the simulation program for determining the minimum value of delay time in excess of the mean. The distribution of production times was normal with the means given and the standard deviation equal to half the mean. A buffer capacity equal to one unit less than that calculated was also used in the simulation program and the minimum delay time found. A one-tailed, t-test was performed on the differences between these two sets of data. A one-tailed test was used because the buffer size of one less than that calculated should produce a higher value of delay time.

The hypothesis to be tested would then be that the mean of the differences $(\overline{X_1 - X_2})$ between samples is zero $(H_0 : \mu = 0)$. The alternate hypothesis is that the mean is greater than zero $(H_1 : \mu > 0)$. The mean of the difference between the samples must be used rather than the means of the two samples because the resulting delay times are correlated. This correlation occurs because both buffer capacities are determined once the calculated value is found.

Using the above method the following values were obtained by simulation:

MEAN (MIN)	CALCULATED BUFFER SIZE (UNITS)	DELAY TIME (MINUTES) X_1	DELAY TIME (MINUTES) X_2	DIFFERENCE BETWEEN SAMPLES
0.521	21	0.010	0.005	0.005
1.610	8	0.007	0.003	0.004
0.132	39	0.014	0.006	0.008
0.091	42	0.013	0.008	0.005
0.147	37	0.010	0.006	0.004
1.009	13	0.009	0.004	0.005
1.988	6	0.005	0.002	0.003
0.435	23	0.012	0.006	0.006
0.089	42	0.014	0.008	0.006
1.213	12	0.008	0.003	0.005

$$\overline{X_1 - X_2} = \overline{0.0051}$$

$$S = \text{Standard Deviation} = \sqrt{1/(n-1)\sum_{i=1}^{n}(d_i - \bar{d})^2} = 0.00137$$

$$t = \frac{(\overline{X_1 - X_2} - 0.)\sqrt{n-1}}{S} = 11.17$$

Confidence Interval	D.O.F.	Value from t-dist.
.90	9	0.906
.95	9	1.131

The calculated value of t is very significant and indicates that a decrease in capacity from the value calculated using the function $y = Ae^{-Bx}$ results in a significant delay time increase. The increase is demonstrated by the one-tailed test with the values X_1 being those utilizing the calculated buffer capacity minus one unit. The alternate hypothesis was then accepted and the X_1 values are greater, in all cases, than those for X_2.

An increase in buffer capacity of one unit did not decrease the delay time from that value found for the calculated capacity.

CHAPTER VI

CONCLUSIONS AND RECOMMENDATIONS

Conclusion

The intention of this paper was to develop a model for determining an optimal capacity, in-process inventory buffer stage to decouple each production stage in a series line. This optimal capacity should provide effective decoupling of production stages, thereby minimizing delay time and maximizing the lines' throughput, with minimum dollar investment in the buffer stages.

The model was developed for a series production line through the use of a simulated model of the line and known distributions of processing times for each stage in the line. From the simulation results obtained it was concluded that the distribution of production times and length of the line (number of production stages in the line) have no significant effect on buffer capacity. Only the mean output time per unit is required to determine the optimal buffer capacity.

The buffer capacity was found to be related to output time per unit in the following manner:

$$y = 3.251\ e^{-0.086x}$$

where, y = output time per unit of production

x = optimal buffer capacity.

Using this relationship, for any given output rate (reciprocal of y), the optimum capacity for buffer stages can be determined. The calculated capacity can be used for any value of unit output time below 2.5 minutes. Above this value, the optimum capacity approaches unity. The model is then limited to the output rate of 24 per hour or higher.

To demonstrate that the developed model does give the smallest buffer capacity which results in minimum delay, random samples of means were obtained and the minimum unit delay determined by simulation. Then, the buffer capacity was reduced by one unit and delay time was again determined. A one-tailed, t-test was performed on the mean difference between the two sets of data. Results of this test indicate a very significant increase in delay time. The same test was performed with the buffer capacity being increased by one unit and results indicated that the decrease in delay time was not significant. From this it is concluded that the minimum delay point was reached.

On a cost basis, to determine that this optimum capacity (resulting in minimum delay) is economically feasible, comparison should be made between cost of maintaining this inventory and savings realized by better machine utilization. If a savings results, the buffer should be used.

When applying this model, a particular problem must be investigated for defining characteristics before application is attempted. All results stated assume a balanced line, equal capacity buffers between stages and buffers between each production stage.

Recommendations for Further Study

Some obvious areas for further study arise from results presented in this paper.

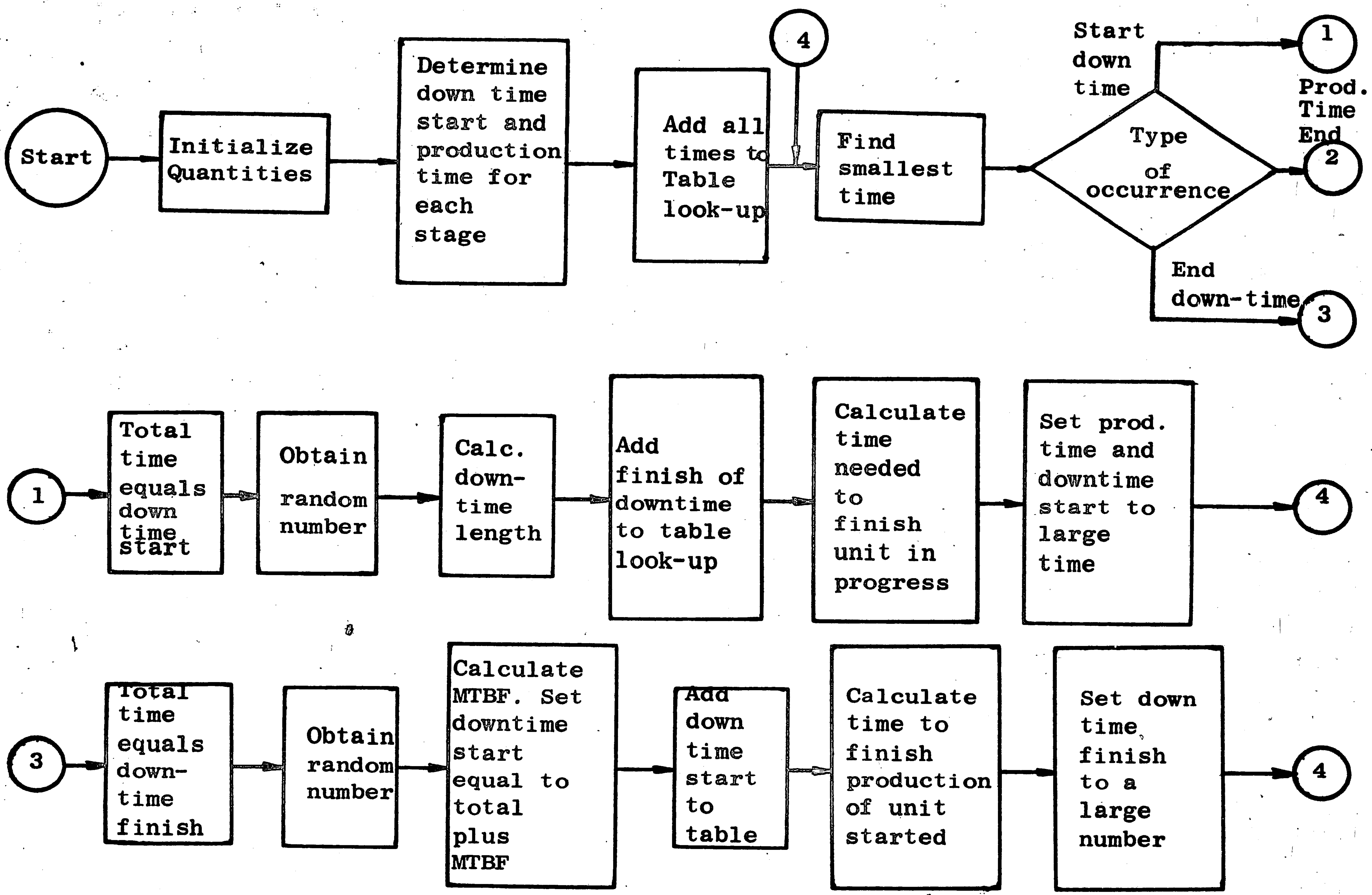
First, the assumption of equal buffer capacities between all stages may not be necessary. It was noticed during the investigation that no difference in buffer capacity was noticed for a 5 or 10 stage line. In other words, the greatest buffering probably takes place in the first few stages. Some reduction in capacity as a function of the stage can probably be made.

Another area which should be investigated is whether or not a buffer should be included between every stage in the line. It may be possible that only every other stage or some other combination would prove optimal. Here the investigation would be most productive if applied to specific problems since a general statement could probably not be made.

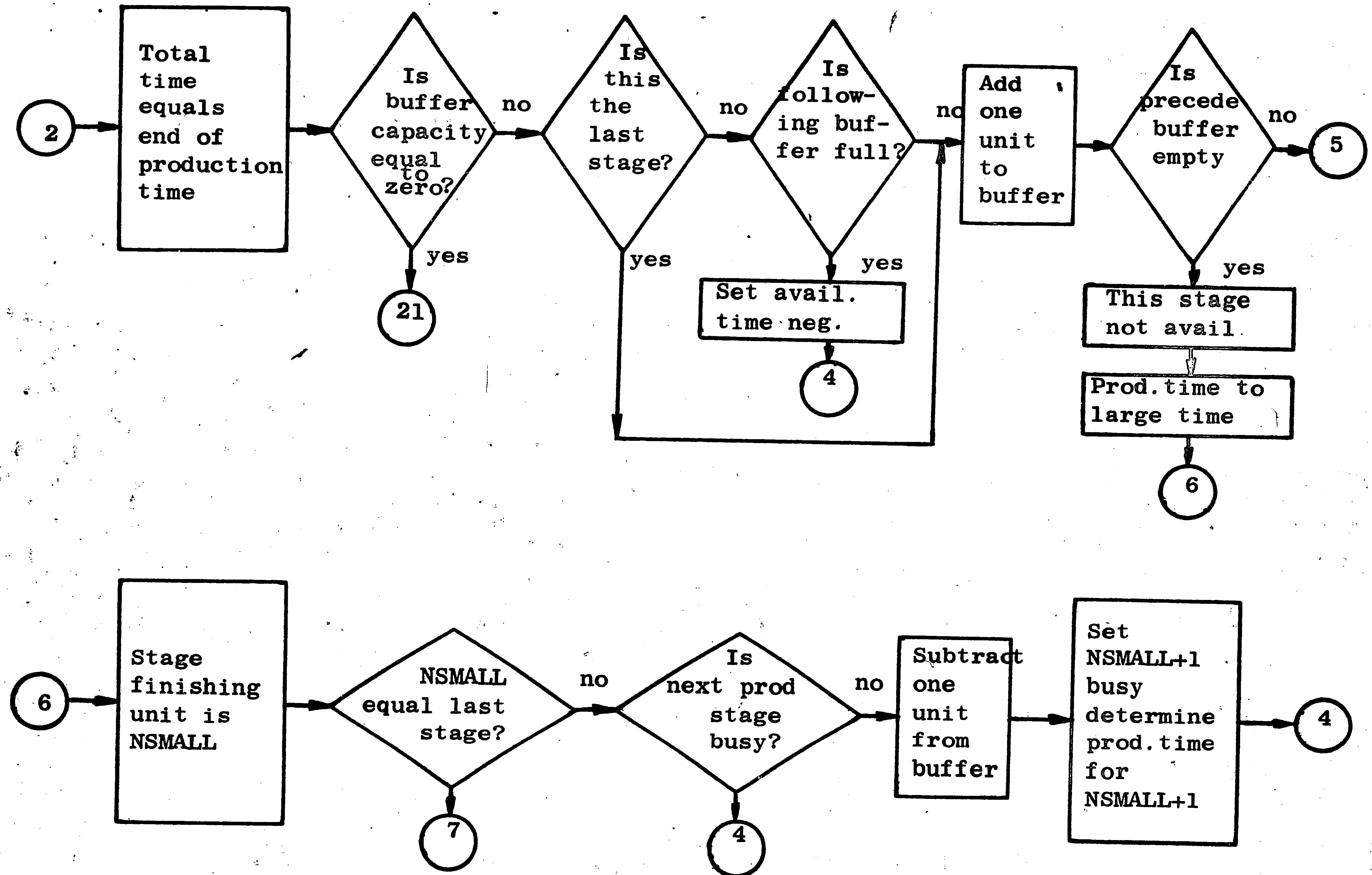
Finally, while the assumption of a balanced line is a necessary one, the effect of parallel stages in a line could be investigated to determine whether one buffer should supply both stages or possibly allow space for separate buffers. This type of arrangement could affect the buffer capacities required.

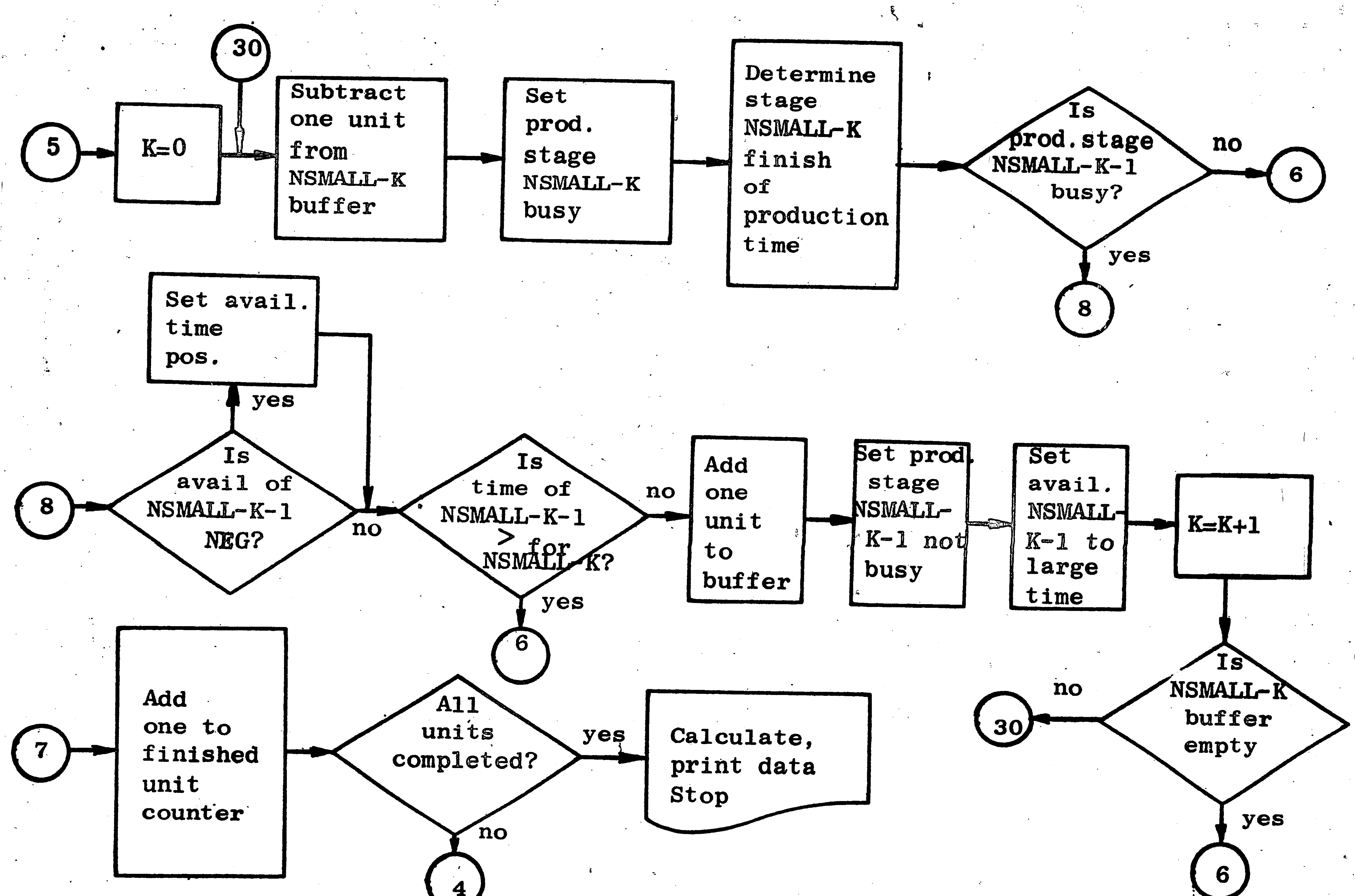
Appendix A

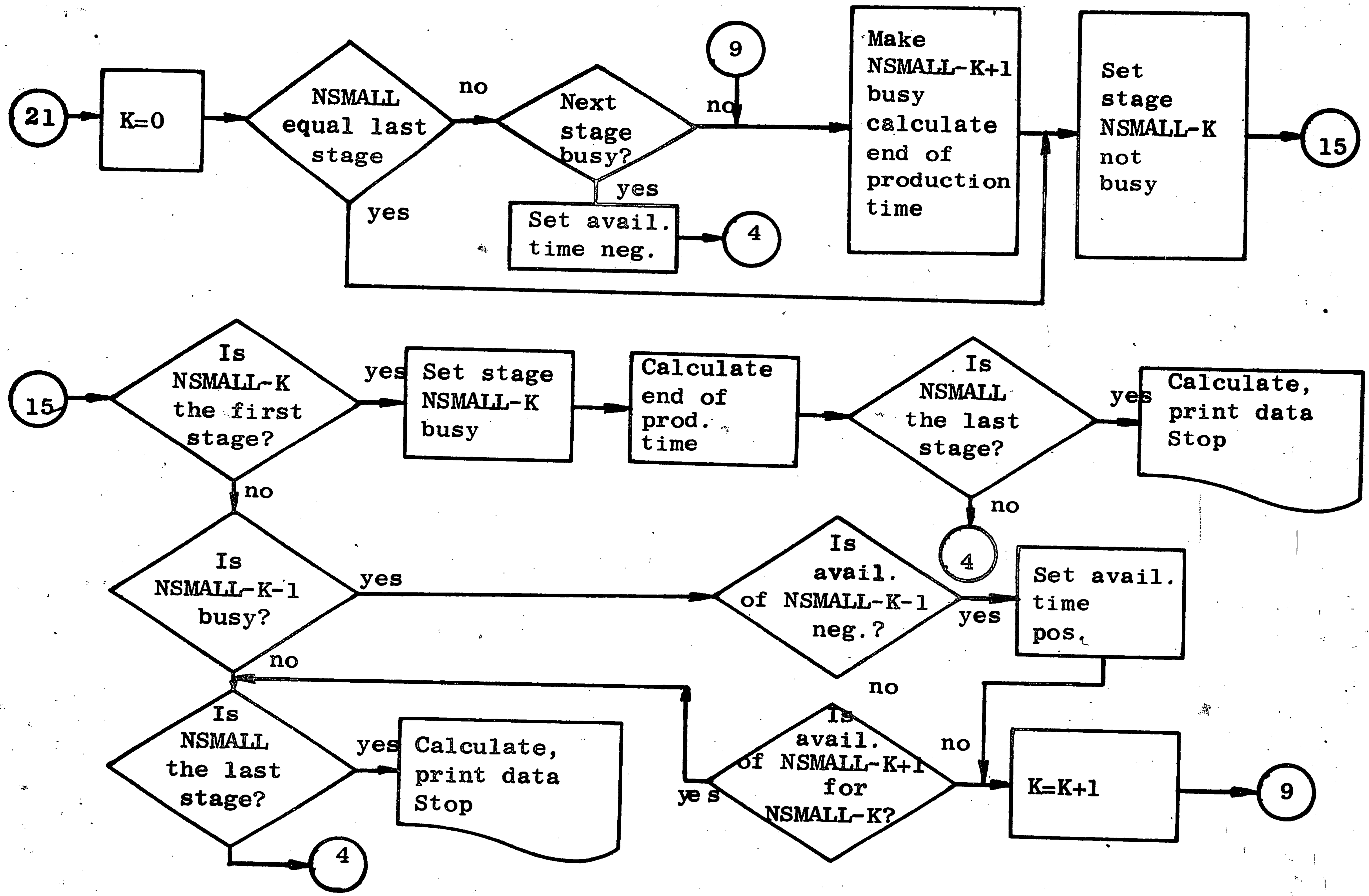
Simulation Program Flow Chart



Start
Initialize Quantities
Determine down time start and production time for each stage
Add all times to Table look-up
4
Find smallest time
Type of occurrence
Start down time
1
Prod. Time End
2
End down-time
3
1
Total time equals down time start
Obtain random number
Calc. down-time length
Add finish of downtime to table look-up
Calculate time needed to finish unit in progress
Set prod. time and downtime start to large time
4
3
Total time equals down-time finish
Obtain random number
Calculate MTBF. Set downtime start equal to total plus MTBF
Add down time start to table
Calculate time to finish production of unit started
Set down time, finish to a large number
4


2
Total time equals end of production time
Is buffer capacity equal to zero?
no
yes
21
Is this the last stage?
no
yes
Is follow-ing buf-fer full?
no
yes
Set avail. time neg.
4
Add one unit to buffer
Is precede buffer empty
no
5
yes
This stage not avail.
Prod.time to large time
6
6
Stage finishing unit is NSMALL
NSMALL equal last stage?
no
7
Is next prod stage busy?
no
4
Subtract one unit from buffer
Set NSMALL+1 busy determine prod.time for NSMALL+1
4


5
K=0
30
Subtract one unit from NSMALL-K buffer
Set prod. stage NSMALL-K busy
Determine stage NSMALL-K finish of production time
Is prod. stage NSMALL-K-1 busy?
no
6
yes
8
Set avail. time pos.
yes
8
Is avail of NSMALL-K-1 NEG?
no
Is time of NSMALL-K-1 > for NSMALL-K?
no
yes
6
Add one unit to buffer
Set prod. stage NSMALL-K-1 not busy
Set avail. NSMALL-K-1 to large time
K=K+1
Is NSMALL-K buffer empty
no
30
yes
6
7
Add one to finished unit counter
All units completed?
yes
Calculate, print data Stop
no
4


21
K=0
NSMALL equal last stage
no
yes
Next stage busy?
no
yes
9
Set avail. time neg.
4
Make NSMALL-K+1 busy calculate end of production time
Set stage NSMALL-K not busy
15
15
Is NSMALL-K the first stage?
yes
no
Set stage NSMALL-K busy
Calculate end of prod. time
Is NSMALL the last stage?
yes
no
Calculate, print data Stop
4
Is NSMALL-K-1 busy?
yes
no
Is avail. of NSMALL-K-1 neg.?
yes
no
Set avail. time pos.
Is NSMALL the last stage?
yes
Calculate, print data Stop
4
Is avail. of NSMALL-K+1 for NSMALL-K?
yes
no
K=K+1
9

Appendix B

Mathematical Description of the Simulation Program

MODEL DEVELOPMENT

(1) $T_{n,i} = R_{n,i} + I_{n,i}$

(2) where $I_{n,i} = [T_{n-1,i} - P_{n,i-1}] \cdot P(E_n^i)$

(3) but $T_{n-1,i} = R_{n-1,i} + I_{n-1,i}$

(4) so $I_{n,i} = [R_{n-1,i} + I_{n-1,i} - P_{n,i-1}] \cdot P(E^i)$.

(5) Since $I_{n-1,i} = [T_{n-2,i} - P_{n-1,i-1}] \cdot P(E_{n-1}^i)$,

substitute (5) into (4) and obtain,

(6) $I_{n,i} = [R_{n-1} + (T_{n-2,i} - P_{n-1,i-1}) \cdot P(E_{n-1}^i) - P_{n,i-1}] \cdot P(E_n^i)$.

(7) $T_{n-2,i} = R_{n-2,i} + I_{n-2,i} - R_{n-2,i} + (T_{n-3,i} - P_{n-2,i-1}) \cdot P(E_n^i)$.

Substituting (7) into (6) and obtain

(8)
$$I_{n,i} = [R_{n-1,i} + \{(R_{n-2,i} + [T_{n-3,i} - P_{n-2,i-1}] \cdot P(E_{n-2}^i)) - P_{n-1,i-1}\} \cdot P(E_{n-1}^i) - P_{n,i-1}]P(E_n^i) .$$

Carrying this process further we finally come to the following equation:

(9)
$$I_{n,i} = \sum_{J=1}^{n-1} \left[R_{n-J,i} \cdot \prod_{L=1}^{J} P(E_{n-L+1}^{L^i}) \right] - \sum_{J=0}^{n-2} [P_{n-J,i-1}] \prod_{L=0}^{J} P(E_{n-L}^i)$$

$$(10) \quad I_{n,i} = \sum_{J=1}^{n-1} D_{n-J} \cdot \prod_{L=1}^{J} P(E^{i}_{n-L+1})$$

$$+ \sum_{J=0}^{n-2} (P_{n-J-1,i} - P_{n-J,i-1}) \cdot \prod_{L=0}^{J} P(E^{i}_{n-L})$$

but (11) $\displaystyle\sum_{J=1}^{n-1} D_{n-J,i} \cdot \prod_{L=1}^{J} P(E^{i}_{n-L+1})$

$$= \sum_{J=1}^{n-1} [T_{n-J+1,i-1} - P_{n-J,i}] \cdot P(F^{i}_{n-J+1}) \cdot \prod_{L=1}^{J} P(E^{i}_{n-L+1}),$$

since $D_{n-J,i} = (T_{n-J+1,i-1} - P_{n-J,i}) \cdot P(F^{i}_{n-J+1})$.

Equation (11) is equal to zero since

$$P(F^{i}_{n-J+1}) \cdot \prod_{L=1}^{J} P(E^{i}_{n-L+1}) = 0.$$

This condition exists because buffer n-L+1 cannot be both empty and full at the time the ith unit arrives at that bunker. The probability of both occurrences is zero since they are mutually exclusive events.

Therefore:

$$I_{n,i} = \sum_{J=1}^{n-1} (P_{n-J,i} - P_{n-J+1,i-1}) \cdot \prod_{L=1}^{J} P(E^{i}_{n-L+1})$$

Now an expression for $R_{n,i}$ must be derived.

(1) $R_{n,i} = P_{n,i}+D_{n,i}$

$$= P_{n,i}+[T_{n+1,i-1}-P_{n,i}]\ .\ P(F^{i}_{n+1})$$

(2) $T_{n+1,i-1} = R_{n+1,i-1}+I_{n+1,i-1}$

(3) $R_{n,i} = P_{n,i} + [R_{n+1,i-1}+I_{n+1,i-1}-P_{n,i}] \cdot P(F^{i}_{n+1})$

but $R_{n+1,i-1} = P_{n+1,i-1}+ (T_{n+2,i-2}-P_{n+1,i-1})\ P(F^{i-1}_{n+2})$

and $T_{n+2,i-2} = R_{n+2,i-2}+I_{n+2,i-2}$, so:

(4) $R_{n,i} = P_{n,i} + [I_{n+1,i-1}-P_{n,i}]\ \ P(F^{i}_{n+1})$

$$+ [P_{n+1,i-1}+ \left\{R_{n+2,i-2}+I_{n-2,i-2}-P_{n+1,i-1}\right\}]P(F^{i-1}_{n+2}) \cdot P(F^{i}_{n+1})\ .$$

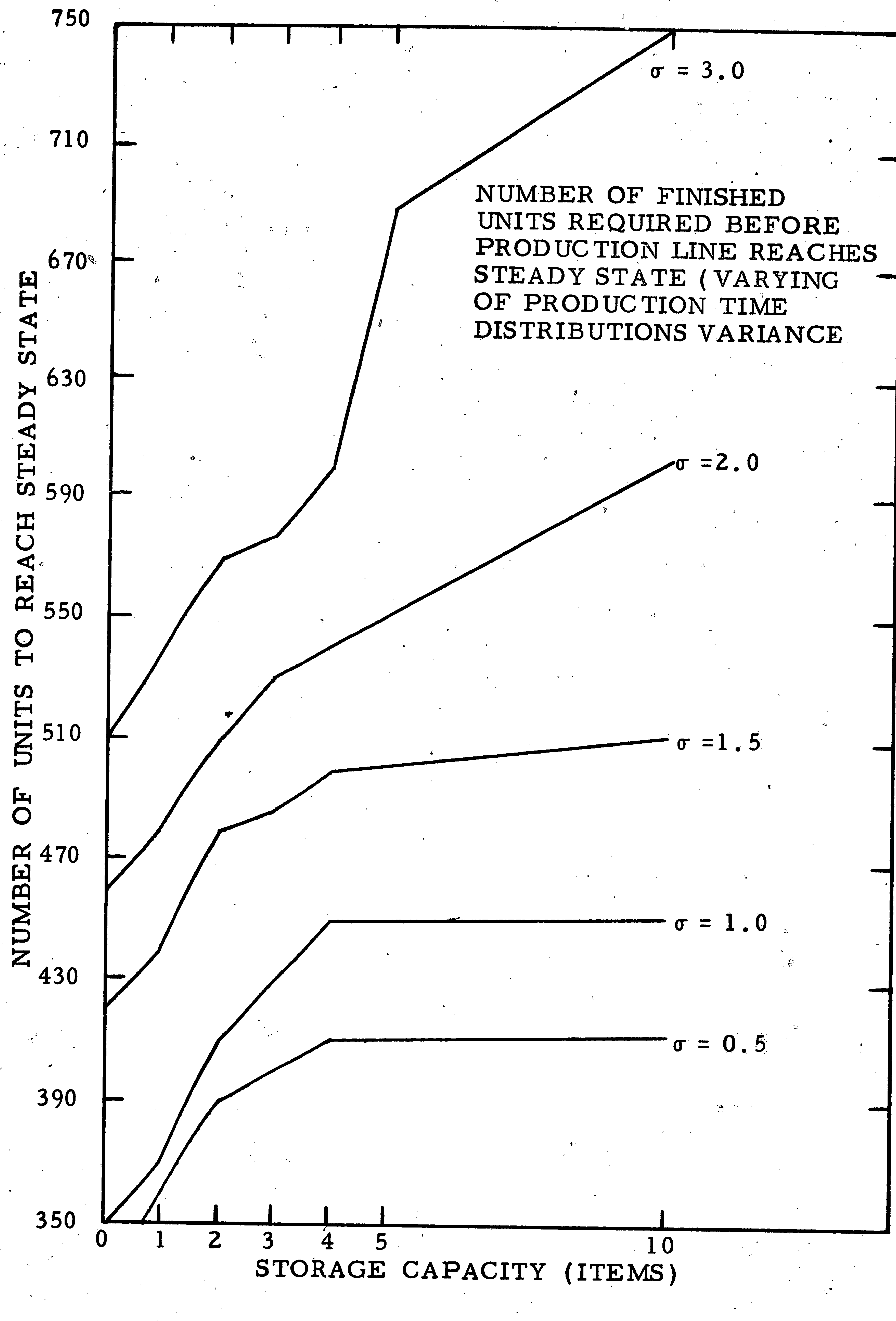
Carrying the substitutions for R and T, the following form is found for $R_{n,i}$:

(5) $$R_{n,i} = P_{n,i}+ \sum_{J=n}^{J=N-1} (P_{J+1,i-J+n-1}-P_{J,i-J+n}) \cdot \prod_{L=n}^{J} P(F^{i-L+n}_{L+1})$$

$$+ \sum_{J=n}^{J=N} I_{J+1,i-J+n-1} \cdot \prod_{L=n}^{J} P(F^{i-L+n}_{L+1})$$

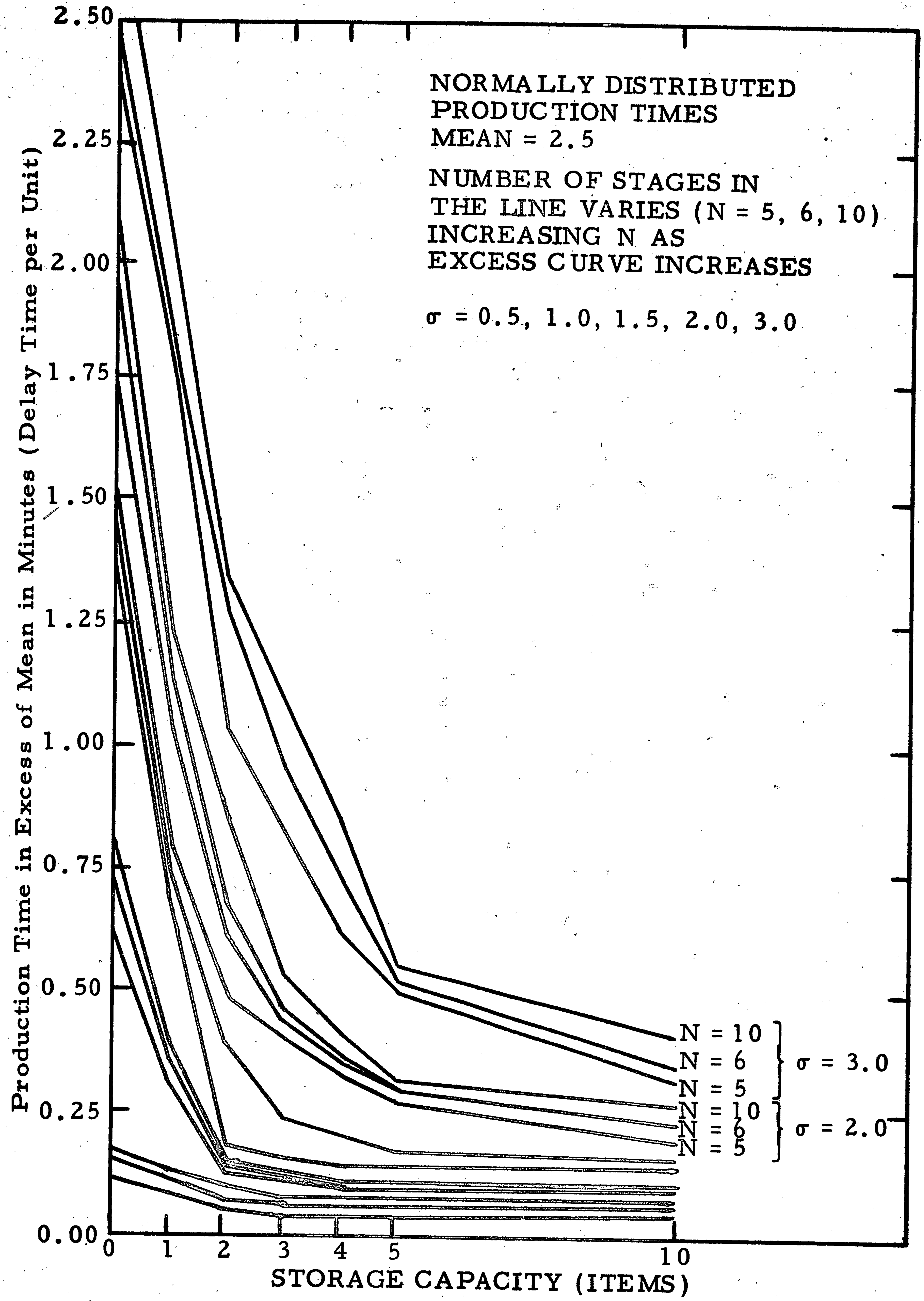
Appendix C

Graphs of Number of Units Required To Reach Steady State


NUMBER OF FINISHED UNITS REQUIRED BEFORE PRODUCTION LINE REACHES STEADY STATE (VARYING OF PRODUCTION TIME DISTRIBUTIONS VARIANCE
NUMBER OF UNITS TO REACH STEADY STATE
STORAGE CAPACITY (ITEMS)
σ = 3.0
σ = 2.0
σ = 1.5
σ = 1.0
σ = 0.5
750
710
670
630
590
550
510
470
430
390
350
0
1
2
3
4
5
10

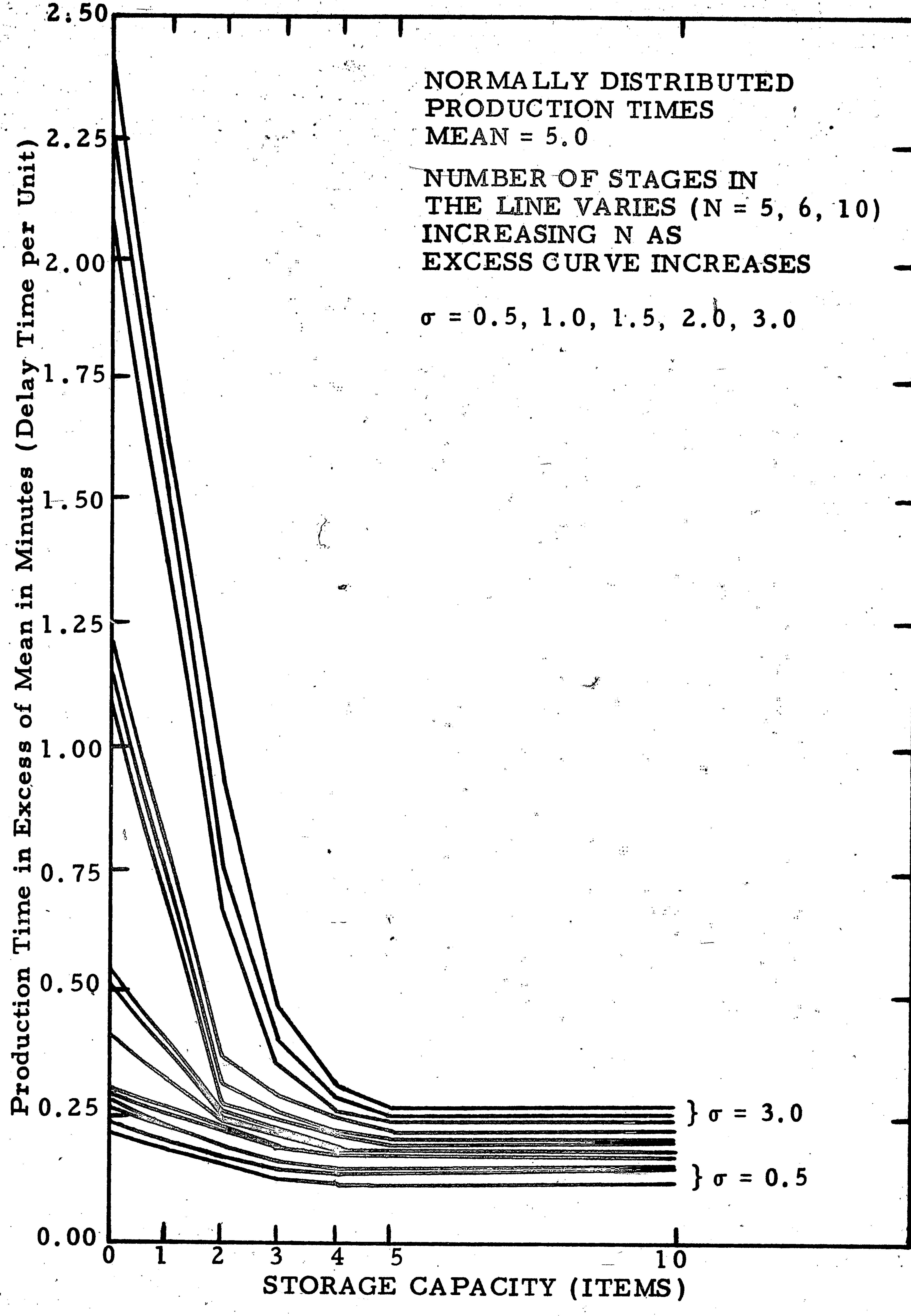
Appendix D

Graphs - Output Time Per Unit
Versus Buffer Capacity-Normal Distribution

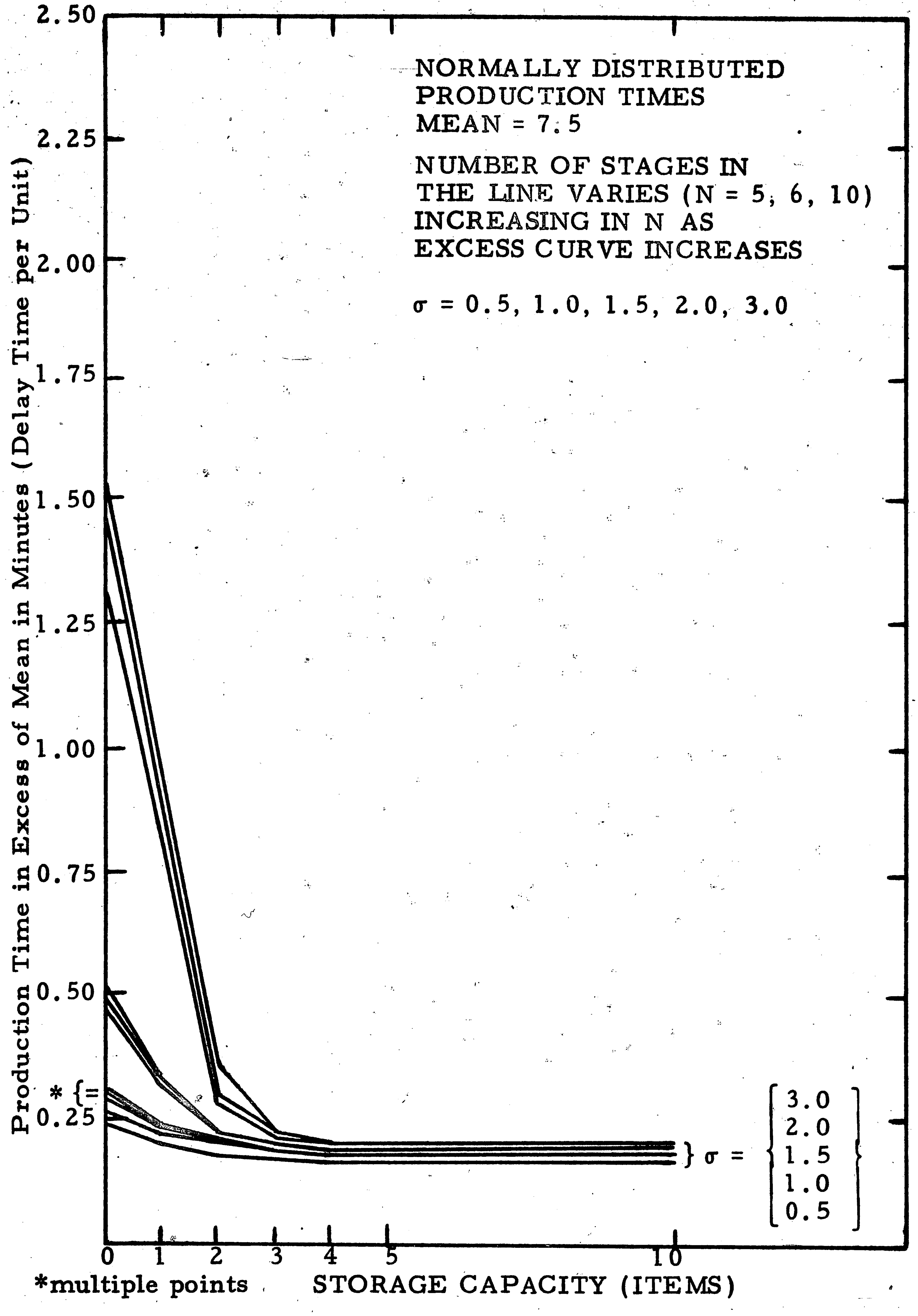


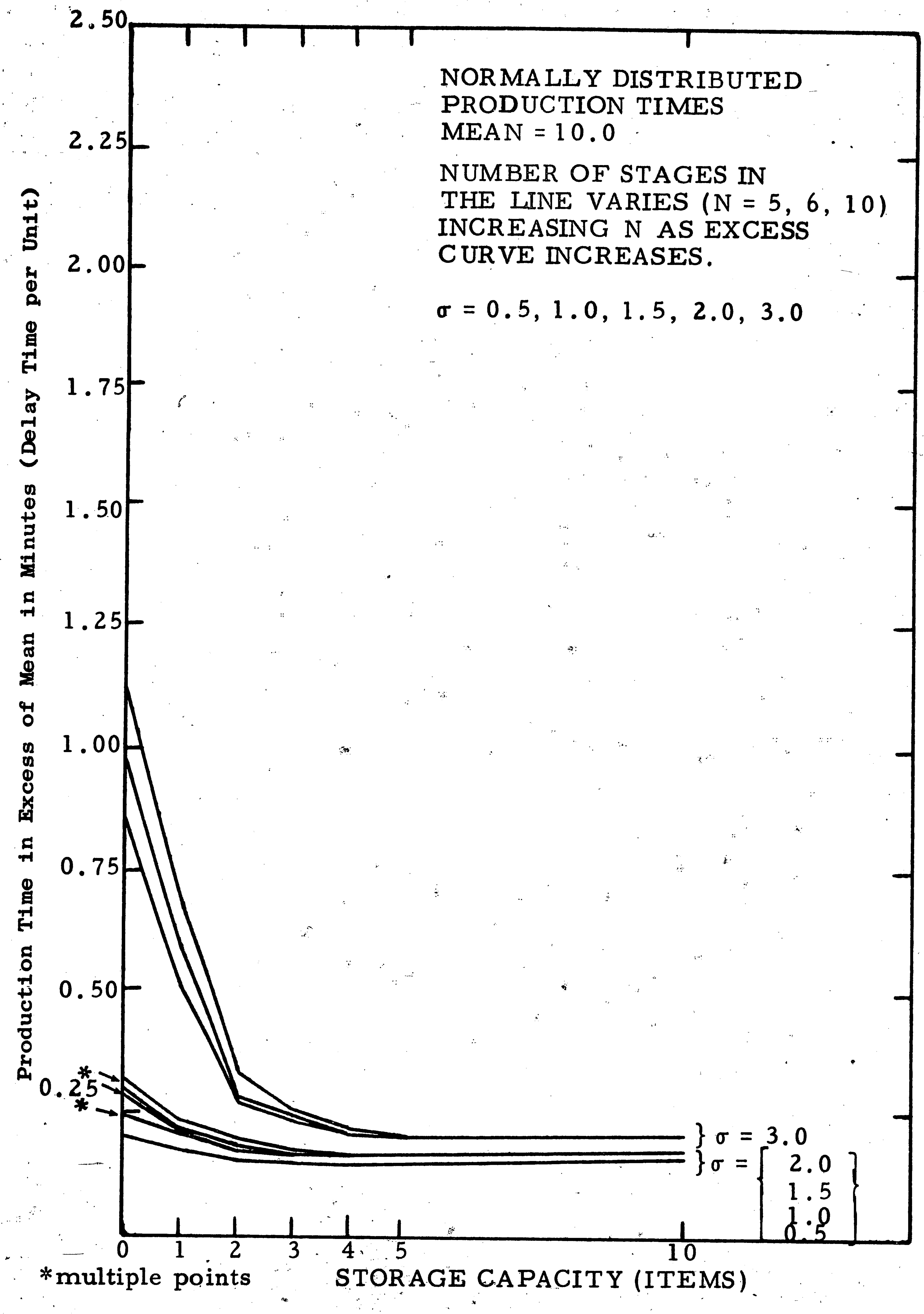
NORMALLY DISTRIBUTED
PRODUCTION TIMES
MEAN = 2.5
NUMBER OF STAGES IN
THE LINE VARIES (N = 5, 6, 10)
INCREASING N AS
EXCESS CURVE INCREASES
σ = 0.5, 1.0, 1.5, 2.0, 3.0
Production Time in Excess of Mean in Minutes (Delay Time per Unit)
2.50
2.25
2.00
1.75
1.50
1.25
1.00
0.75
0.50
0.25
0.00
0
1
2
3
4
5
10
STORAGE CAPACITY (ITEMS)
N = 10
N = 6
N = 5
σ = 3.0
N = 10
N = 6
N = 5
σ = 2.0

D-1


NORMALLY DISTRIBUTED
PRODUCTION TIMES
MEAN = 5.0
NUMBER OF STAGES IN
THE LINE VARIES (N = 5, 6, 10)
INCREASING N AS
EXCESS CURVE INCREASES
σ = 0.5, 1.0, 1.5, 2.0, 3.0
} σ = 3.0
} σ = 0.5
Production Time in Excess of Mean in Minutes (Delay Time per Unit)
2.50
2.25
2.00
1.75
1.50
1.25
1.00
0.75
0.50
0.25
0.00
0
1
2
3
4
5
10
STORAGE CAPACITY (ITEMS)

D-2

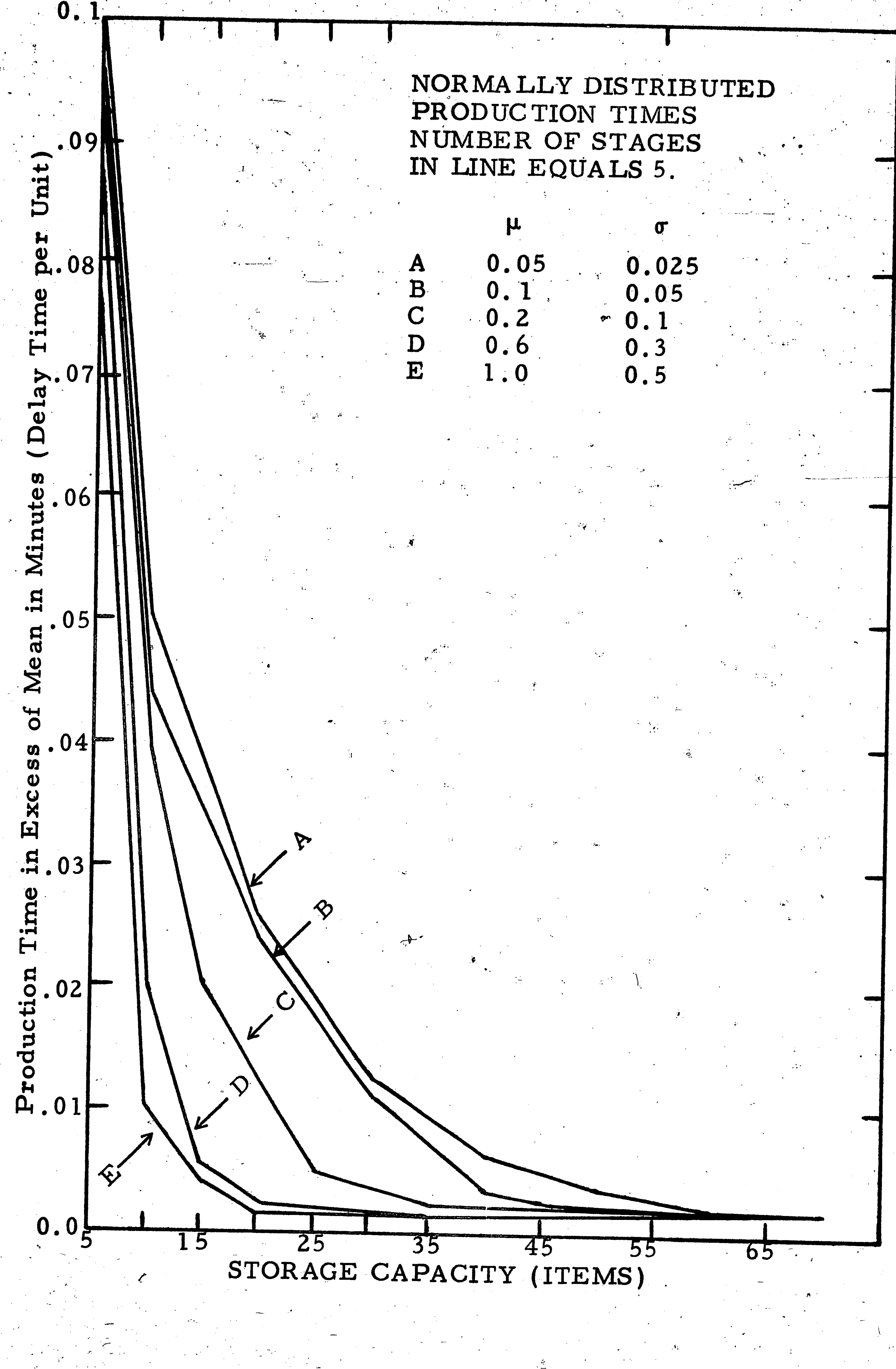

NORMALLY DISTRIBUTED
PRODUCTION TIMES
MEAN = 7.5
NUMBER OF STAGES IN
THE LINE VARIES (N = 5, 6, 10)
INCREASING IN N AS
EXCESS CURVE INCREASES
σ = 0.5, 1.0, 1.5, 2.0, 3.0
Production Time in Excess of Mean in Minutes (Delay Time per Unit)
2.50
2.25
2.00
1.75
1.50
1.25
1.00
0.75
0.50
0.25
* {=
σ = 3.0 2.0 1.5 1.0 0.5
0 1 2 3 4 5 10
*multiple points
STORAGE CAPACITY (ITEMS)


NORMALLY DISTRIBUTED
PRODUCTION TIMES
MEAN = 10.0
NUMBER OF STAGES IN
THE LINE VARIES (N = 5, 6, 10)
INCREASING N AS EXCESS
CURVE INCREASES.
σ = 0.5, 1.0, 1.5, 2.0, 3.0
Production Time in Excess of Mean in Minutes (Delay Time per Unit)
2.50
2.25
2.00
1.75
1.50
1.25
1.00
0.75
0.50
0.25
*
*
} σ = 3.0
} σ = 2.0 1.5 1.0 0.5
0
1
2
3
4
5
10
*multiple points
STORAGE CAPACITY (ITEMS)

D-4

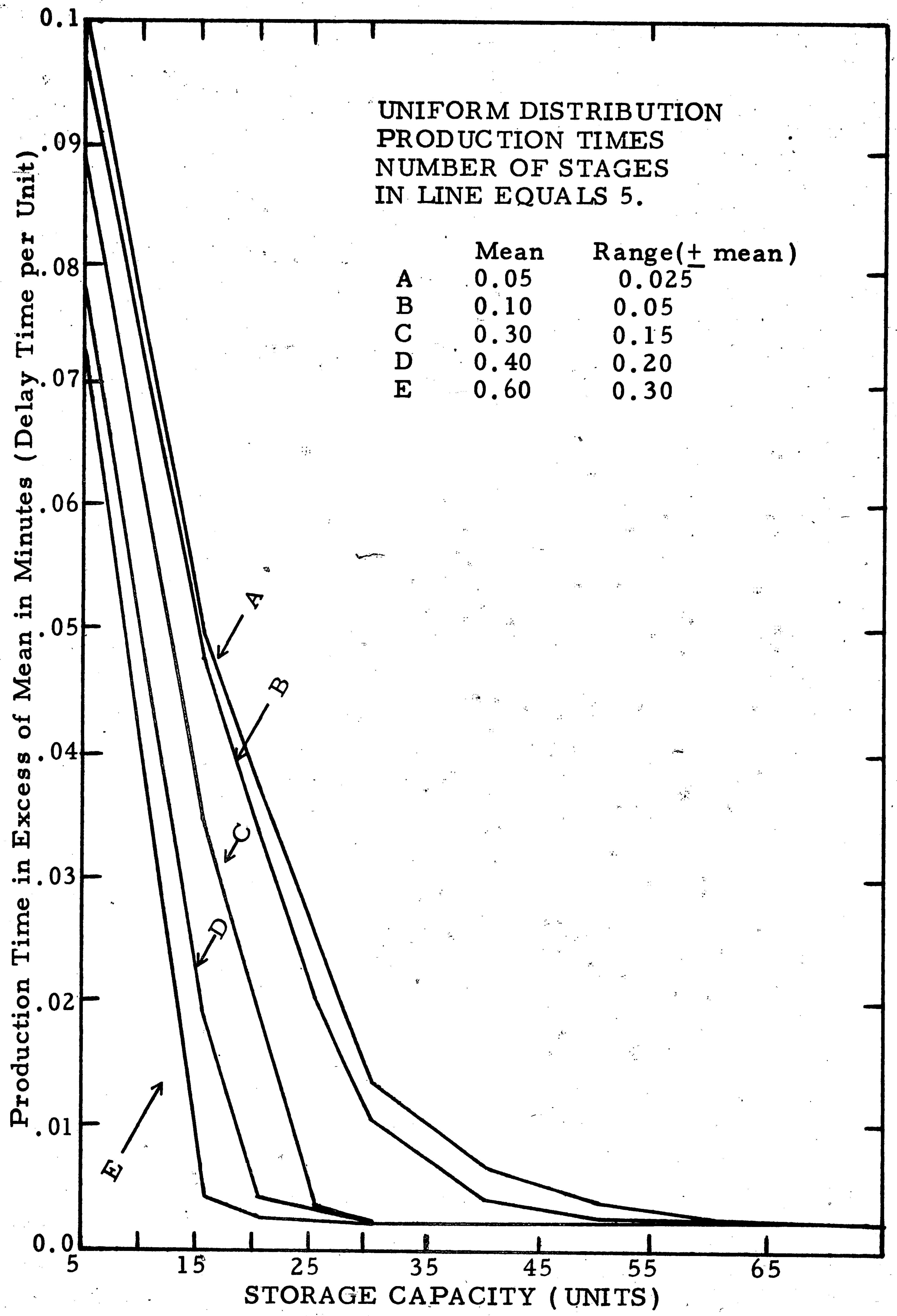
Appendix E

Graphs - Output Time Per Unit
Versus Buffer Capacity

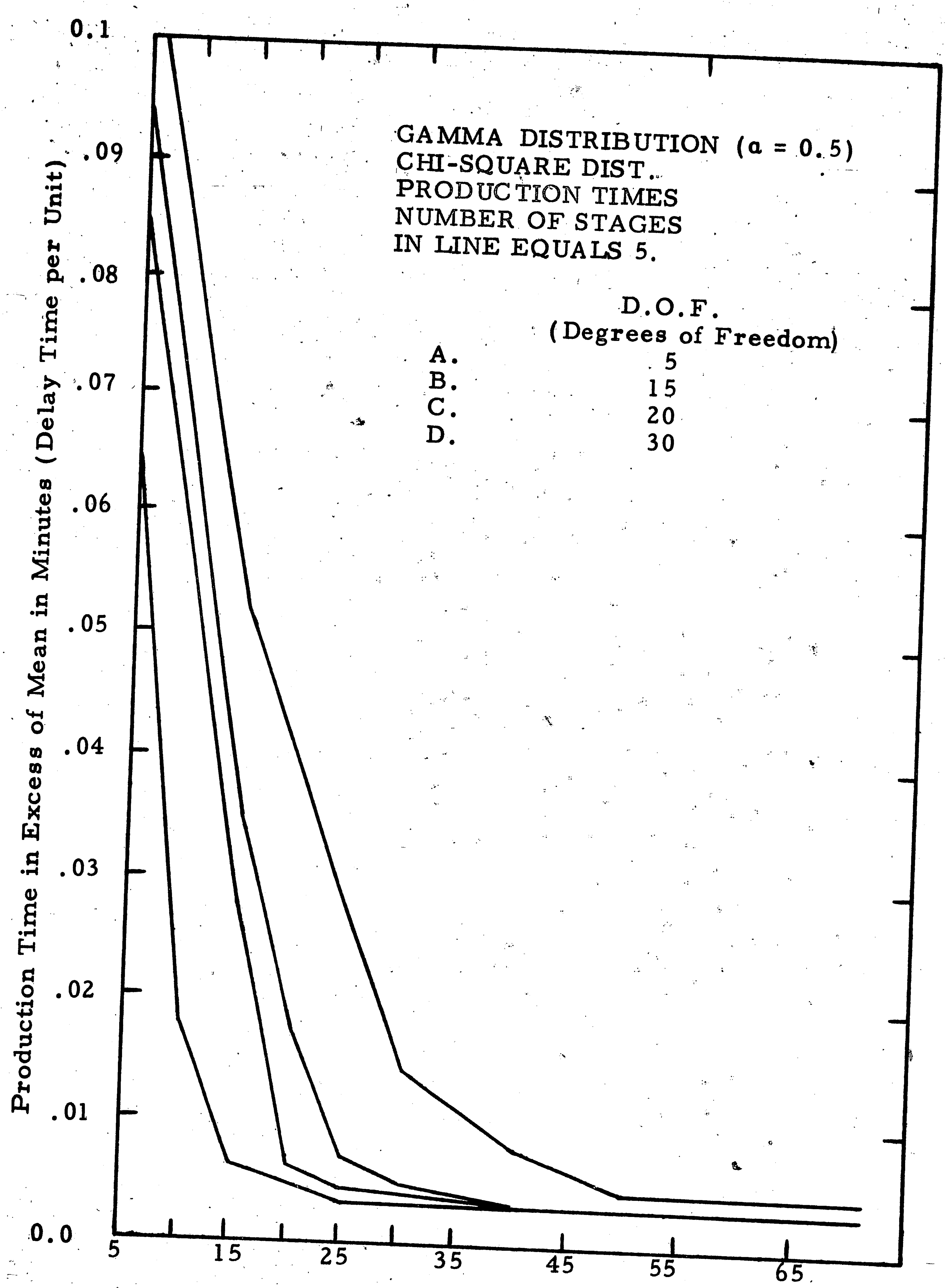


NORMALLY DISTRIBUTED
PRODUCTION TIMES
NUMBER OF STAGES
IN LINE EQUALS 5.
μ σ
A 0.05 0.025
B 0.1 0.05
C 0.2 0.1
D 0.6 0.3
E 1.0 0.5
A
B
C
D
E
Production Time in Excess of Mean in Minutes (Delay Time per Unit)
0.1
.09
.08
.07
.06
.05
.04
.03
.02
.01
0.0
5
15
25
35
45
55
65
STORAGE CAPACITY (ITEMS)

E-1


UNIFORM DISTRIBUTION
PRODUCTION TIMES
NUMBER OF STAGES
IN LINE EQUALS 5.
Mean Range(± mean)
A 0.05 0.025
B 0.10 0.05
C 0.30 0.15
D 0.40 0.20
E 0.60 0.30
A
B
C
D
E
Production Time in Excess of Mean in Minutes (Delay Time per Unit)
0.1
.09
.08
.07
.06
.05
.04
.03
.02
.01
0.0
5
15
25
35
45
55
65
STORAGE CAPACITY (UNITS)

E-2


GAMMA DISTRIBUTION (a = 0.5)
CHI-SQUARE DIST.
PRODUCTION TIMES
NUMBER OF STAGES
IN LINE EQUALS 5.
D.O.F.
(Degrees of Freedom)
A. 5
B. 15
C. 20
D. 30
Production Time in Excess of Mean in Minutes (Delay Time per Unit)
0.1
.09
.08
.07
.06
.05
.04
.03
.02
.01
0.0
5
15
25
35
45
55
65

E-3

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